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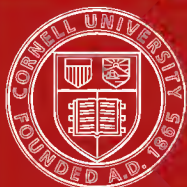
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HEAT
AN ELEMENTARY TEXT-BOOK
THEORETICAL AND PRACTICAL

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CAMBRIDGE NATURAL SCIENCE MANUALS
PHYSICAL SERIES.

HEAT

AN ELEMENTARY TEXT-BOOK
THEORETICAL AND PRACTICAL

FOR COLLEGES AND SCHOOLS.

BY

R. T. GLAZEBROOK, M.A., F.R.S.

ASSISTANT DIRECTOR OF THE CAVENDISH LABORATORY,
FELLOW OF TRINITY COLLEGE, CAMBRIDGE.

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PREFACE.

IT has now come to be generally recognised that the most satisfactory method of teaching the Natural Sciences is by experiments which can be performed by the learners themselves. In consequence many teachers have arranged for their pupils courses of practical instruction designed to illustrate the fundamental principles of the subject they teach. The portions of the following book designated **EXPERIMENTS** have for the most part been in use for some time as a Practical Course for Medical Students at the Cavendish Laboratory.

The rest of the book contains the explanation of the theory of those experiments, and an account of the deductions from them; these have formed my lectures to the same class. It has been my object in the lectures to avoid elaborate apparatus and to make the whole as simple as possible. Most of the lecture experiments are performed with the apparatus which is afterwards used by the class, and as far as possible the theoretical consequences are deduced from the results of these experiments.

In order to deal with classes of considerable size it is necessary to multiply the apparatus to a large extent. The students usually work in pairs and each pair has a separate

table. On this table are placed all the apparatus for the experiments which are to be performed. Thus for a class of 20 there would be 10 tables and 10 specimens of each of the pieces of apparatus. With some of the more elaborate experiments this plan is not possible. For them the class is taken in groups of 5 or 6, the demonstrator in charge performs the necessary operations and makes the observations, the class work out the results for themselves.

It is with the hope of extending some such system as this in Colleges and Schools that I have undertaken the publication of the present book and others which are to follow. My own experience has shewn the advantages of such a plan, and I know that that experience is shared by other teachers. The practical work interests the student. The apparatus required is simple; much of it might be made with a little assistance by the pupils themselves. Any good-sized room will serve as the Laboratory. Gas should be laid on to each table, and there should be a convenient water supply accessible; no other special preparation is necessary.

The plan of the book will, I hope, be sufficiently clear; the subject-matter of the various Sections is indicated by the headings in Clarendon type; the Experiments to be performed by the pupils are shewn thus:

EXPERIMENT (1). *To determine the fixed points of a thermometer.*

These are numbered consecutively. Occasionally an account of additional experiments, to be performed with the same apparatus, is added in small type. Besides this the small-type articles contain some numerical examples worked out, and, in many cases, a notice of the

principal sources of error in the experiments, with indications of the method of making the necessary corrections. These latter portions may often with advantage be omitted on first reading. A few articles of a more advanced character, which may also at first be omitted, are marked with an asterisk.

A book which has grown out of the notes in general use in a laboratory is necessarily a composite production. I have specially to thank Mr Wilberforce and Mr Fitzpatrick for their help in arranging many of the experiments. Mr Fitzpatrick has also given me very valuable assistance by reading the proofs and suggesting numerous improvements. The illustrations have for the most part been drawn from the apparatus used in the class by Mr Hayles, the Lecture Room Assistant, and Mr E. Wilson.

R. T. GLAZEBROOK.

CAVENDISH LABORATORY,

January 1, 1894.

Arrangements have been made with Messrs F. E. BECKER AND Co. 33, 35 and 37 Hatton Wall, E.C. to supply the apparatus required to perform the EXPERIMENTS described in the following pages. Price Lists may be had on application to them.

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HEAT.

CHAPTER I.

HEAT AND ENERGY.

1. The Nature of Heat. When we stand in the sunshine or in front of the fire it feels hot. If we take hold of some ice or snow it feels cold. Heat is the name given to the cause of these and the like sensations. In the first case heat enters our body, in the second it leaves it and our sensations make us aware of its transference. What then is the Nature of Heat?

This question can be more fully answered when we have studied some of the effects which Heat causes and some of the methods by which Heat can be produced. We shall then be able to appreciate the meaning of the statement that,

Heat is one of the forms in which Energy becomes known to us.

We will however at once consider this statement a little more in detail.

2. Work and Energy. If a body, under the action of a force, moves in the direction in which the force acts, work is done on the body.

Thus, when a man lifts a weight, he applies force to it and does work; when a cannon ball penetrates a target it exerts force on the target, and does work, being itself stopped in the process.

The capacity that a body or system of bodies has for doing work is called Energy.

Thus the statement that heat is one of the forms of energy implies that heat is one form which the capacity of a body for doing work may take.

Now a body may have Energy because of its position relative to other bodies and of the forces which act on it. Energy in this form is called Potential; a stone at the top of a cliff, the weight of a clock which has just been wound up, or the coiled mainspring of a watch, all possess potential energy.

Again a moving body possesses energy; a falling stone can do work. Energy in this form is called Kinetic.

***3. Transformation of Energy.** Energy can change from kinetic to potential or vice versa.

A stone at the top of a cliff has no kinetic energy; relative to the cliff its energy is all potential. As the stone falls it loses potential energy, for this is proportional to its height above the earth, being measured by mgz where m is the mass of the stone, z its height and g the acceleration due to gravity; at the same time it gains kinetic energy, for this is measured by $\frac{1}{2}mv^2$, where v is the velocity, and this increases till the stone reaches the ground. Moreover, we can shew that, for a falling body, the gain in kinetic energy is equal to the loss of potential energy, for if z be the height when the velocity is v and h the height from which the stone started we have from Dynamics

$$\frac{1}{2}mv^2 = mg(h - z),$$

since $h - z$ is the space in which a body moving with uniform acceleration g has acquired a velocity v ; now $\frac{1}{2}mv^2$ is the gain of kinetic energy, while $mg(h - z)$ is the loss of potential energy; thus these are equal.

In this case and in many others we meet with in Mechanics, there is neither gain nor loss of energy, it is simply changed from the potential to the kinetic form.

4. Apparent loss of Energy. When the stone has reached the ground it has apparently lost its energy, it has no potential energy, for it can fall no further; it has no kinetic energy, for it is at rest. Careful observation would shew

however that another change has taken place; the stone has been heated.

Or again, a railway train in motion even after the steam has been shut off has a large store of kinetic energy. When the brakes are put on and the train brought to rest this kinetic energy disappears as such, but here again heat has been produced, in this case, by the friction.

5. Heat and Work. Various experiments shew us that there is some connection between heat and work. Thus, take a strip of lead and bend it backwards and forwards or hammer it; in both cases work is done and it will be found that the lead is warmed. Or again attach a brass tube *AB* fig. (1) to a whirling table so that it can be made to rotate rapidly round a vertical axis by turning the flywheel *C*. Fill

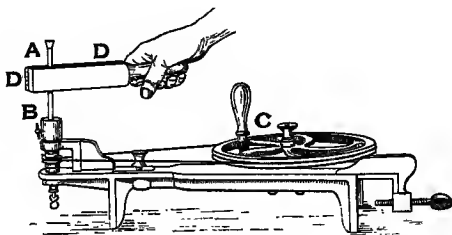


Fig. 1.

the tube with water and close it with a cork; on turning the wheel the tube can be easily made to rotate rapidly. *DD* are two pieces of wood in each of which there is a semicircular groove; these are united by means of a hinge and can be made to clip the tube tightly. On doing this there is considerable friction produced; more energy is needed to turn the wheel; the water becomes heated, and finally boils. Work has been transformed into Heat.

Now though in the experiments just described, it would be impossible to measure with accuracy the work done and the heat produced, we can in various ways arrange experiments in which these measures can be made.

6. Joule's Experiments. This was first done satisfactorily by Dr Joule of Manchester in 1843, and it is to his experiments, which lasted over many years, that most of our knowledge of the connection between heat and work is due.

In one investigation he employed the following arrangement. A known mass of water is contained in a vertical brass cylinder. Within this a shaft carrying paddles revolves. Vanes are secured to the sides of the cylinder and the moving paddles whirl between them. The friction between the paddles and the water produced heat. By observing with a thermometer the temperature of the water, the heat produced can be calculated (see Section 176).

A wooden cylinder is attached to the shaft and two thin strings are coiled in opposite directions round the cylinder. The strings pass over two pulleys and carry known weights. On releasing the shaft the weights fall, turning the paddles and stirring the water. The energy lost by the weights in the fall can be calculated and thus the amount of energy required to produce a given quantity of heat can be determined.

Joule found as the result of these and similar experiments that the amount of energy required to produce a definite quantity of heat is itself a definite quantity; in order to produce the unit quantity of heat that is to raise 1 gram of water 1° Centigrade, 41.9 million ergs of work are needed.

7. The Conservation of Energy. Thus in the case just described, the potential energy of the weights is not lost, it is transformed into heat; the weights, the water and other substances which have been heated, possess the same amount of energy at the end as at the beginning; only the distribution of the energy has been changed; the weights have less, the water more.

The same is true of the falling stone; the potential energy it possessed on the cliff, has been changed, firstly into the form of the kinetic energy of the visible mass and then into heat in the stone itself and in the ground where it struck; some has been used to produce the noise of the blow, some in heating slightly the air through which the stone has fallen; the sum total of all these different forms remains the same.

Again conversely, Energy in the Heat form can be transformed into Mechanical Work. The steam engine is a familiar instance of a machine for this purpose. Part of the heat produced by the combustion of the fuel in the furnace passes into the boiler and thence with the steam into the cylinder; as the steam expands in the cylinder it loses heat, and work is done by the engine.

In this case the practical difficulties in the way of measuring the exact amount of heat which is used in doing the external work are very great. Still when heat is turned into work and due allowance is made for all the losses which take place it is found that the work produced is always proportional to the heat producing it—one unit of heat, if it can be converted entirely into work, will produce 41.9 million ergs. It is true that in order to convert the heat entirely into work certain other conditions have to be fulfilled, but we need not attempt to discuss these at present. We are thus justified in stating that Heat is a form of Energy.

***8. The Nature of Heat-energy.** It is a further question, and one we can go but a little way towards answering if we ask, What is the nature of heat-energy? Let us consider it briefly.

We know that all bodies are made up of a large number of particles called molecules. These molecules are extremely small, still we have now a fairly accurate idea of their size. In all substances they are in a state of rapid agitation, vibrating backwards and forwards, colliding with each other and with the sides of the vessel containing the substance.

In a gas the molecules are much less closely packed than in a liquid or a solid. It is only when they come very close together that they exert any force on each other; for the greater part of their existence they are free from each other's action; in consequence of their motion they possess kinetic energy and it is practically certain that the heat of the gas is almost entirely the kinetic energy of agitation of its molecules. When the gas is heated the molecules are made to vibrate more rapidly, they gain kinetic energy and the gain is a measure of the heat supplied.

To some extent a similar statement is true also of a solid, the molecules are in a state of vibration and heat increases the energy of this vibration, but the molecules are also on the average much closer together than in a gas. Each molecule is acted on by forces due to its neighbours and thus possesses potential energy; the action of heat may modify these forces and so alter the potential, as well as the kinetic, energy of the molecules. Thus we can say very little about the form in which heat-energy exists in a solid or a liquid; in a gas we know that it is in the main the kinetic energy of agitation of the molecules, in a solid it is in all probability in part the kinetic energy of the molecules, in part their potential energy arising from the forces they exert upon each other.

***9. Historical Account of the principle of the Conservation of Energy.** As we have said our knowledge of the true nature of heat dates from the experiments of Joule in 1843, still it had been foreshadowed at a much earlier time. Lord Bacon in the *Novum Organon* states his belief that Heat consists in a kind of motion or "brisk agitation" of the parts of a body. Robert Boyle one of the original members of the Royal Society expressed the same opinion, so also did John Locke; but the first experiments from which the truth might have been deduced were those of Benjamin Thompson, Count Rumford, who in 1799 produced sufficient heat by friction to raise about 27 lbs. of water from the freezing to the boiling point, and of Sir Humphrey Davy who in the previous year had shewn that ice could be melted by friction only. Rumford's experiments give us enough data to calculate the mechanical equivalent of heat; he omitted however one experiment which was necessary to make his reasoning conclusive. Davy failed to draw correct conclusions from his work.

Séguin in 1837 and Mayer in 1842 calculated the equivalent; they both however made assumptions which though true needed experiment for their justification. This experiment was performed by Joule in 1845.

CHAPTER II.

TEMPERATURE AND ITS MEASUREMENT.

10. Effects of Heat. When heat is applied to bodies it produces the following among other effects :

- (1) Change of dimensions or of volume.
- (2) Change of internal stress.
- (3) Change of state.
- (4) Change of temperature.
- (5) Electrical and chemical effects.

We will briefly consider each of these in turn.

(1) *Change of dimensions.*

Most bodies expand or increase in volume on being heated. In laying down the rails on a line of railway an interval is left between consecutive rails to allow for this. The tyre of a wheel is put on red hot, as it cools it contracts and binds the wheel tightly together. Boiler plates are riveted with red-hot rivets for the same reason. The pendulums or balance wheels of clocks and watches require compensation, otherwise each change of temperature would cause a variation in the clock's rate. The ocean currents and the trade winds are due to the change in volume, and therefore of density, of the water or air produced by heat.

(2) *Change of internal stress.*

Many of these changes of volume are accompanied by changes in the stress or internal forces between the molecules

of the body. As the wheel tyre contracts it is subject to great force. Walls of buildings have been drawn together by passing iron bars through them, heating the bars and screwing on nuts to the projecting ends; as the bars contract they draw the walls together. An air balloon placed in front of a fire expands, the pressure which the contained air exerts on the india-rubber covering increases and bursts the covering.

(3) *Change of state.*

Many substances can exist in the three states of bodies—solid, liquid, or gaseous,—changing from one to the other on the application or withdrawal of heat. A lump of ice melts when sufficient heat is applied and becomes water; apply more heat, the water becomes warmer, and after a time it boils, being converted into steam. Ice, water, steam, are all different forms of the same chemical compound of oxygen and hydrogen; the application of heat, among other effects, may change the arrangement of the molecules of the substance and the forces between them and thus convert ice into water or water into steam.

(4) *Change of temperature.*

Place the hand in a basin of cold water. It feels cold: apply heat to the water, it gradually becomes warmer; in scientific language its “temperature” is said to rise. Or again, put a red-hot poker into a vessel of water, the poker is cooled, the water heated, heat passes from the poker to the water, the temperature of the poker is lowered, that of the water increased. In both these cases there is a transference of heat from one body to the other, the body from which the heat passes is said to have the higher temperature.

11. Definition of Temperature. *Temperature is the condition of a body on which its power of communicating heat to or receiving heat from other bodies depends.*

If when two bodies *A* and *B* are put into thermal communication, heat passes from *A* to *B*, then *A* is said to be at a higher temperature than *B*.

Two bodies *A* and *B* have the same temperature, if when they are put into thermal communication there is no transference of heat between them.

The flow of heat from one body to a second depends on the difference between the temperatures of the two; the body at the lower temperature may originally possess more heat than the other, but heat will pass into it raising its temperature and reducing that of the hotter body. The total amount of heat in a bucket of water may very likely be greater than that in a red-hot poker; still the poker is at the higher temperature and is thereby able to communicate some of its heat-energy to the water at a lower temperature. The temperature determines the direction of the flow.

Temperature is analogous to level or pressure in hydrostatics. Consider two reservoirs *A*, *B* connected by a pipe with a tap. On opening the tap, water may flow from *A* to *B* or vice versa, or there may be no flow at all. The condition which determines which of these alternatives takes place is the difference of level between the surfaces of the water in *A* and *B*. If the level in *A* is above that in *B* water flows from *A* to *B* and vice versa. Water from a reservoir on a hill side, even though it be small in quantity, runs down to the sea because the reservoir is above the level of the sea.

Or again, take a vessel containing compressed air and open the stop-cock; the air rushes out into the atmosphere, where the pressure is less, until the pressures within and without the vessel are equalized. The temperature of the hot body corresponds to the pressure of the air, the air flows from places of high to places of low pressure. Heat passes from a body at a high temperature to one at a lower temperature.

Temperature is also analogous to potential in the science of electricity. Positive electricity flows from places of high potential to places of low potential.

12. Comparison of Temperatures. Experience shews us that while our ideas of temperature and of the difference between two temperatures may in the first instance be derived from our sense of touch, no accurate knowledge can be obtained from that source alone.

If a piece of metal and a piece of flannel which are lying

side by side in front of the fire be touched, the metal will appear hotter than the flannel though the two may be shewn by a suitable experiment to be at the same temperature; if on the other hand the two be very cold the metal will appear the colder; the sensation does not depend only on the temperature but also on the rate at which heat is transferred through the metal or the flannel.

We require some other method of measuring temperatures. We must for this purpose employ one of the other effects produced by heat on matter; the effect usually chosen is the dilatation or increase of volume of a liquid. Suppose we have two bodies *A* and *B* and we wish to determine which of the two is at the higher temperature. We take a third body *C* the change in volume of which we can easily measure; the mercury or alcohol in an ordinary thermometer is such a body¹. When heat passes into the thermometer, raising its temperature, the liquid expands. Place it in contact with *A*. Heat passes into it and the column of mercury rises until the temperature of the thermometer is the same as that of *A*. Note the height of the column. Now place the thermometer in contact with *B*. If *B* is at a higher temperature than *A* more heat will pass into the thermometer and the column will rise still further, if *B* is at a lower temperature the column will fall.

We can thus determine which of the two *A* or *B* is at the higher temperature.

We cannot however yet say whether this difference is greater or less than that between two other bodies *D* and *E*, neither can we compare a temperature observed in one place with another observation of temperature made elsewhere unless we can transport the same thermometer. For such purposes we need a scale of temperature.

13. The fixed points on a thermometer. The

¹ This thermometer consists of a glass tube of fine bore terminating in a bulb. The tube and part of the bulb are filled with mercury (or alcohol) the rest of the tube being empty. Small changes in the volume of the mercury in the bulb shew themselves by a considerable motion of the end of the column in the narrow tube. We shall explain later the mode of filling and testing a thermometer.

temperature at which ice melts is found to be always the same at ordinary pressure¹.

The temperature of steam issuing from boiling water is also constant when the pressure is constant.

These two facts are verified by experiments which we shall describe later, for the present we are considering how we may use them to construct a scale of temperature. They give us two fixed points which can be marked on the stem of a thermometer; when the mercury stands at the lower of the points the thermometer is at the temperature of melting ice, when the mercury is at the upper mark the temperature is that of steam at a certain standard pressure.

14. Scales of Temperature. The difference in temperature between these two fixed points is very considerable; we need some means of subdividing it so that we may compare any two temperatures more closely than would be possible if we only had the two fixed points of reference. As the temperature rises from the freezing point to the boiling point the mercury in a glass thermometer expands by a certain definite amount.

We define as a rise of temperature of one degree, that rise of temperature which causes the mercury to expand by some definite fraction of the total expansion between the freezing and the boiling points.

There are three scales of temperature in more or less common use.

(1) The Fahrenheit Scale. The number of degrees between the two fixed points is 180. Thus the temperature changes by 1° Fahr. when the volume of the mercury of a mercurial thermometer alters by $\frac{1}{180}$ th part of the total increase between freezing point and boiling point. On Fahrenheit's Scale the freezing point is marked 32° ; the boiling point is thus $32^{\circ} + 180^{\circ}$ or 212° .

¹ A small change is produced by variation of pressure but it is too small to affect the determination of the lower fixed point.

(2) **The Centigrade Scale.** The number of degrees between the two fixed points is 100. Thus the temperature changes by 1° C. where the volume of the mercury in a mercurial thermometer alters by $\frac{1}{100}$ th part of the total increase between the freezing point and the boiling point. On the Centigrade Scale freezing point is marked 0° , boiling point 100° .

(3) **Réaumur's Scale.** The number of degrees between the two fixed points is 80. Thus the temperature changes by 1° Réaumur where the volume of the mercury in a mercurial thermometer alters by $\frac{1}{80}$ th part of the total increase between the freezing and the boiling point. On Réaumur's Scale, freezing point is marked 0° , boiling point 80° .

Fahrenheit's Scale is in common use in this country for ordinary purposes while Réaumur's is still similarly employed on the continent. The Centigrade Scale has now been very generally adopted for scientific work.

15. Comparison of Scales of Temperature. To compare the three scales consider a thermometer to which all three are fitted, fig. 2. Let *A* be the freezing point, *B* the boiling point, *P* the position of the end of the mercury column, and let *F*, *C*, *R* be the readings on the three scales respectively corresponding to the point *P*.

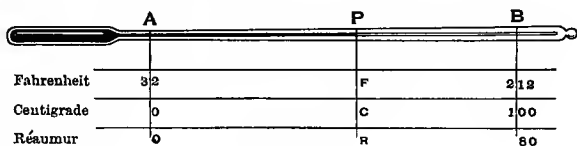


Fig. 2.

Now on the three scales respectively the distance *AP* represents *F* - 32, *C*, and *R* degrees, while *AB* represents 180 (212 - 32), 100, and 80 degrees. But *AP* is the same fraction of *AB*, whichever scale be employed.

Thus the fractions $\frac{F-32^1}{180}$, $\frac{C}{100}$ and $\frac{R}{80}$ are all equal ;

$$\therefore \frac{F-32}{180} = \frac{C}{100} = \frac{R}{80}.$$

and from these expressions the relations between the scales can be found.

It must be noticed that for Fahrenheit readings below zero Fahrenheit, we must treat F as negative, and, in order to find the number of degrees between freezing point and the reading, we must add 32 to the reading instead of subtracting as explained above. The corresponding Centigrade reading will of course be negative.

Examples. (1) Find the Centigrade and Reaumur readings corresponding to 60° Fah.

60° Fah. is 60 - 32 or 28° above freezing point.

$$1^\circ \text{ Fah.} = \frac{5}{9} \text{ of } 1^\circ \text{ C.}$$

$$\therefore 28^\circ \text{ Fah.} = 28 \times \frac{5}{9} \text{ C.} = 15^\circ \cdot 55 \text{ C.}$$

$$1^\circ \text{ Fah.} = \frac{4}{9} \text{ of } 1^\circ \text{ R.}$$

$$\therefore 28^\circ \text{ Fah.} = 28 \times \frac{4}{9} \text{ R.} = 12^\circ \cdot 4 \text{ R.}$$

Or, using the formula, and denoting by C. and R. the Centigrade and Reaumur readings corresponding to 60° Fah. we have

$$\text{C.} = \frac{5}{9} (60 - 32) = 15^\circ \cdot 55.$$

$$\text{R.} = \frac{4}{9} (60 - 32) = 12^\circ \cdot 4.$$

(2) Find the Fahrenheit reading corresponding to 18° C.

The number of Fah. degrees in 18 degrees C. is $\frac{9}{5}$ of 18, or 32·4.

\therefore the Fahrenheit reading is 32°·4 + 32° or 64°·4.

Or, from the formula,

$$\text{F.} = 32^\circ + \frac{9}{5} \text{ C.} = 32^\circ + 32^\circ \cdot 4 = 64^\circ \cdot 4.$$

(3) The freezing point of mercury is given by the same numbers on the Centigrade and Fahrenheit scale, find this temperature.

If C. and F. are corresponding readings on the two scales, then

$$\text{F.} = \frac{9}{5} \text{ C.} + 32.$$

But by the question, F. and C. are to be represented by the same number, X suppose, then

$$X = \frac{9}{5} X + 32.$$

$$\therefore \frac{4}{5} X = -32.$$

$$\therefore X = -40.$$

¹ It must be noted that F is the Fahrenheit reading corresponding to a Centigrade reading C , not the number of degrees Fahrenheit which are equal to C degrees Centigrade, this last number is $F - 32$.

16. Electrical and chemical effects due to heat.

The most important electrical effect due to heat for our purpose is that discovered by Seebeck who found that if a circuit be composed of different materials, two wires say of iron and copper joined together at each end, and if the temperature of the two junctions be different then an electric current is produced round the circuit. Since a very small electric current can be measured easily this fact is made use of in the Thermopile to measure small differences of temperature.

Heat also produces an important effect in changing the electrical resistance of bodies. This has been made the basis of a method of measuring high temperatures.

Certain crystals also shew electrical effects when heated.

Many chemical effects require heat for their production. Thus the combustion of coal is due to the combination of the carbon of the coal with the oxygen of the air. This combination only takes place at a high temperature. Heat is therefore required to raise the temperature and start the action which, when once started, produces sufficient heat for its continuance.

17. Sources of Heat. Among the sources of heat available for our use we may reckon

1. The Sun.
2. Chemical action.
3. Mechanical action.
4. Electric currents.
5. Change of physical state.
6. The internal heat of the earth.

(1) *The Sun.* Of the above the Sun is by far the most important. We shall have to study at a later stage how the Sun's heat reaches us in the form of radiant energy as it is called.

Directly or indirectly the sun is the source of nearly all the available energy we possess. For our food we are indebted to the sun. Vegetable life depends on sunshine. Our fuel

—coal—is due to the sun's action which in past time enabled plants to decompose the carbonic acid of the air and store up the carbon which we use. The winds and tides, the rainfall which feeds our rivers and is the source of our waterpower, all depend on solar action. Life as we know it would be impossible without the sun.

(2) *Chemical action.* Many chemical actions are accompanied by the production of heat. First among these we may place combustion. When carbon and oxygen unite, a great amount of heat is evolved. Still more is produced by the combination of hydrogen with oxygen as in the oxyhydrogen flame. The heat of our bodies is due to the combination of our food with the oxygen of the air.

(3) *Mechanical sources.* Of these friction is the chief; we have already considered the connection between heat and mechanical work which is transformed into heat by friction.

(4) *Electricity.* When an electric current passes through a conductor it heats it. If the current be sufficiently large and the resistance of the conductor considerable, the rise of temperature may be great. The filaments of incandescent lamps are made to glow by the passage of a current. Water can be boiled and food cooked by heat thus produced.

(5) *Change of physical state.* Just as it requires heat to melt ice, so heat can be obtained by freezing water. The molecules of the substance in the liquid form possess more energy than in the solid; absorb this energy and the liquid becomes a solid. When steam condenses to water, heat is given out. It is possible to cool down various solutions, of which sulphate of sodium is one, below the temperature at which they would normally solidify. If this be done and a small bit of the solid be then dropped into the solution, solidification at once takes place, and is accompanied by a considerable production of heat, and a corresponding rise of temperature.

(6) *The internal heat of the earth.* A large store of heat exists in the interior of the earth but not in a very available form.

18. Historical. The suggestion to use the temperatures of melting ice and steam at a standard pressure for the fixed points is due to Newton (1701). The thermometer was originally invented by Galileo, who employed an air thermometer (Section 23) to determine the temperature of the human body. Soon after his time thermometers containing a liquid in a sealed glass tube were made in Florence for Rinieri, who recorded a number of observations on temperature. These thermometers were lost on the suppression of the Accademia del Cimento, but discovered in 1829. Their scale was then compared with that of Réaumur, and Rinieri's old observations regained their scientific value. One of these thermometers was given by the Grand Duke of Tuscany to the late Prof. Babbage. On his death it was presented by his son to the University of Cambridge, and is now in the Cavendish Laboratory.

It appears from the investigations of Dr Gamgee that Fahrenheit in 1714 used as his two fixed points the temperature of a mixture of ice and salt, in definite proportions, which he called 0° , and the temperature of the human body. This he originally marked 24° . In later thermometers he introduced four times as many divisions, marking the temperature of the human body 96° . Having obtained this scale he continued it up the tube, and found that water boiled at 212° and froze at 32° . These then were taken as more convenient fixed points and more accurate observation has proved that if boiling point is called 212° , the normal temperature of the body is $98^{\circ}\cdot4$, instead of 96° , as originally taken by Fahrenheit.

CHAPTER III.

THERMOMETRY.

19. Construction of a Mercury Thermometer.

A bulb *A* is blown at one end of a glass tube of narrow uniform bore. A cup or funnel *B* is formed at the other end¹. At *C* a short distance below the funnel the tube is drawn out by heating it in a blow-pipe flame so as to form a narrow neck. This is for the purpose of sealing off the thermometer when made.

If mercury be poured into *B* it will not run down the tube to fill the bulb, the bore is too narrow; the following method is therefore adopted. A small quantity of mercury is placed in *B* and the bulb is gently heated; the air expands and some of it bubbles out through the mercury in *B*. The bulb is then allowed to cool and the pressure of the enclosed air falls. The atmospheric pressure forces some of the mercury down the tube and, if sufficient air has been expelled, into the bulb. When this takes place the mercury in the bulb is boiled, the vapour of mercury forces most of the air out of the upper part of the bulb and tube. When the bulb is again cooled the mercury vapour condenses and more mercury flows in from the reservoir. By repeating the process once or twice the last traces of air may be removed, and the bulb and tube filled with mercury.

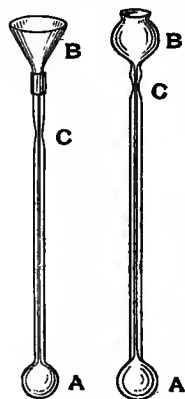


Fig. 3.

¹ In the Laboratory a small funnel may be attached by a piece of india-rubber tubing to the open end, as in the left-hand figure.

Now place the bulb and tube in a bath at a rather higher temperature than the highest at which the thermometer is to be used. Some of the mercury expands into the funnel; remove this and allow the thermometer to cool slowly; the mercury contracts; have a blow-pipe with a small intense flame ready, and as the end of the column is just passing the narrow neck *C*, heat the tube at that point in the flame and draw off the funnel end. By this process the tube is sealed at *C*. The mercury, as it cools, contracts leaving a space filled only with mercury vapour.

Certain precautions are needed before the thermometer can be used. It is found that glass which has been strongly heated continues to change in shape and volume for some time after it has cooled. In consequence of this the mercury rises up the tube, even though the temperature does not change. This process continues for some time; moreover a similar change though smaller in amount takes place whenever the thermometer is heated to a high temperature. By a proper choice of glass for the thermometer tube and by special treatment of the instrument when filled, these defects can be very considerably reduced.

Again, the distance between the fixed marks is usually divided into equal parts. The corresponding portions of the tube will not be equal in volume unless the bore is uniform, and, unless they are equal, the degrees of temperature as given by the thermometer will not really coincide with our definition, for the increase in volume of the mercury for each degree will not be the same. If the tube were as in the figure (fig. 4)—where the want of uniformity is purposely much exaggerated—and if the distances *AB*, *BC*, *CD* etc. be equal, it is clear that the volume between *A* and *B* is larger than that between *C* and *D*; the amount of expansion required to bring the mercury from *A* to *B* is greater than that which will bring it from *C* to *D*, so that the rise of temperature indicated by *AB* is really greater than the rise indicated by *CD*, though nominally the two are the

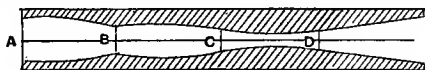


Fig. 4.

same. To avoid this error the tubes of standard thermometers are calibrated. This is done by breaking off from the column a small thread of mercury, placing it by varying the temperature in different parts of the tube and measuring its length. From this measurement the want of uniformity in the tube can be corrected (see Glazebrook and Shaw, *Practical Physics*, p. 89).

Good thermometers for use in scientific work are compared with a standard which has thus been calibrated, and a Table of corrections to the apparent readings is formed.

20. The fixed points. EXPERIMENT (1). *To determine the fixed points of a thermometer.*

(a) The Freezing Point. Wash some ice¹, break it small and pack it round the bulb of the thermometer in a glass or metal funnel (fig. 5), so that the water which forms as the ice melts may drain away into a vessel placed below to receive it. The mercury sinks; heap the ice up round the tube until only the top of the column is visible, and leave it thus for about a quarter of an hour; then with a fine file make a scratch on the glass opposite to the top of the column. This marks the freezing point.

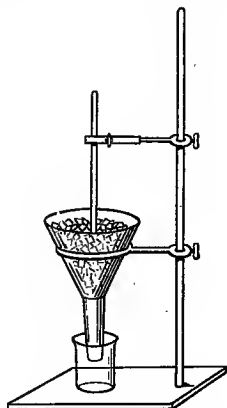


Fig. 5.

(b) The Boiling Point. Place the thermometer in the steam issuing from boiling water. The temperature of the water, if it contain soluble impurities, may differ from that of the steam. In order to secure that the thermometer may be surrounded with steam, a piece of apparatus called an hypsometer is used; a simple form of hypsometer is shewn in fig. 6. A conical tin or copper cover, *BC*, with an inner tube fits loosely on to a glass flask. A cork passes through the top of the tube and the thermometer is inserted through a hole in the cork, its bulb being well above the surface of the water in the flask. As the water boils steam passes round the thermometer bulb and issues between the flask and the loose cover, flowing down on the outside of the flask between it and

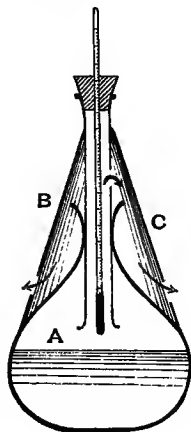


Fig. 6.

¹ The ice obtained in the usual way from a dealer has very often some traces of salt about it which affects the freezing point.

the cover. The thermometer is thus in a current of steam which is protected from the cooling action of the air by the outer downward current. After a time the mercury becomes steady. Adjust the thermometer until the top of the column is just visible above the cork, leave it in that position for a few moments to make sure that the bulb and tube have reached a steady temperature, and then mark the level of the column. Read the height of the barometer; the mark gives the boiling point of water for the pressure at the time of observation. If this be not the standard pressure 760 mm., a correction is required, and this may be obtained as follows.

It has been found that near the standard pressure an increase in pressure due to 26.8 mm. raises the boiling point by 1°C ., and also that for small differences of pressure the change of the boiling point is proportional to the difference in pressure (see § 121). Thus suppose the observed pressure is 752 mm. we have the proportion

$$\frac{\text{required correction}}{1^{\circ}} = \frac{760 - 752}{26.8},$$

$$\therefore \text{required correction} = \frac{8^{\circ}}{26.8} = 0.3.$$

Hence the observed boiling point is 0.3 too low.

Experiments with Thermometers.

(1) Test the fixed points of the given thermometer. The thermometer has been already graduated; determine by the above methods whether the fixed points are correct.

Suppose that in the steam the thermometer reads $99^{\circ}.5$, the height of the barometer being 752 mm. The correction for pressure is, we have seen, $+0.3$. Thus the corrected boiling point is $99^{\circ}.5 + 0.3$ or $99^{\circ}.8$. The upper fixed point therefore has been placed too high by 0.2 .

(2) Mix a little salt with the ice and notice the fall in temperature, then place some salt in the boiling water and observe the boiling point,

immersing the thermometer bulb in the salt and water. It will be too high.

(3) Compare the readings of the two given thermometers. Place the two thermometers (a Fahrenheit and a Centigrade) close together in a water bath, taking care that their bulbs are as near together as possible, and read the two; while taking the readings, stir the water gently and thus secure uniformity of temperature. Let the Centigrade thermometer read 15° . Now since 100 degrees Centigrade are equivalent to 180 degrees Fahrenheit each degree Centigrade is $\frac{9}{5}$ of a degree Fahrenheit. Thus 15° C. are equivalent to $\frac{9}{5} \times 15^{\circ}$ Fah. or 27° Fah. Thus the reading is 27° above freezing point on the Fahrenheit scale, and since freezing point reads 32° Fah. the Fahrenheit reading corresponding to 15° C. is $27^{\circ} + 32^{\circ}$ or 59° Fah.

Assuming the Centigrade thermometer to be correct, determine thus the error of the Fahrenheit instrument.

21. Graduation of a Mercury Thermometer.

When the fixed points are determined the thermometer is graduated by dividing the distance between them into equal parts 180, 100 or 80 according to the scale used. These divisions are continued above the boiling point, and below the freezing point; on the Centigrade and Réaumur scale temperatures below the freezing point are marked as negative. The zero of the Fahrenheit scale is 32° Fah. below freezing point; temperatures below zero are negative.

22. Comparison of the Mercury and other Thermometers. Mercury is not the only liquid used in filling thermometers. Mercury freezes at about -40° C. Alcohol does not freeze till a much lower temperature has been reached, about -130° C., and therefore for very low temperatures alcohol is employed. In some thermometers sulphuric acid is the liquid used.

Such thermometers are graduated by comparison with a mercury thermometer.

The following experiment will explain the reason for this. Suppose we have two thermometers each of uniform bore: the one containing mercury, the other some other liquid, say sulphuric acid. Let the freezing and boiling points of each of the two be determined and marked, and divide the distance between these on each thermometer into 100 parts. Place

the two thermometers side by side in a bath at say 45°C . The column in the mercury thermometer stands at 45° on the scale, that in the sulphuric acid thermometer is not at 45° but at 41° ; if we were to suppose the second thermometer graduated by dividing the distance between the fixed points into equal parts, the temperature as given by it would not be 45° but 41° .

A rise of temperature, which produces in mercury an expansion of $45/100$ of that which occurs between the fixed points, does not produce $45/100$ of the expansion occurring in sulphuric acid between the same fixed points. We might have defined the temperature with reference to the expansion of sulphuric acid; the scale so obtained would differ from that which we have adopted, and therefore the alcohol or sulphuric acid thermometers are graduated by comparison with a standard mercury thermometer. If we experimented with other liquids we should find similar results. Water would give the most anomalous result, for when the mercury thermometer was at 4° the water thermometer would be below zero.

23. The Air Thermometer. For some purposes temperature is measured by the expansion of air or some other gas.

The simplest form of air thermometer would consist of a bulb, fig. 7, with tube attached filled with dry air. The end of the tube is open and a small pellet of mercury or sulphuric acid separates the air in the bulb from the external atmosphere. As the temperature rises the air in the bulb expands, driving the pellet of acid before it; as it falls the air contracts; the position of the pellet indicates the volume of the enclosed air. Now this volume depends in part on the temperature and in part on the pressure of the surrounding air. If the pressure varies, the temperature remaining the same, the pellet will move. In

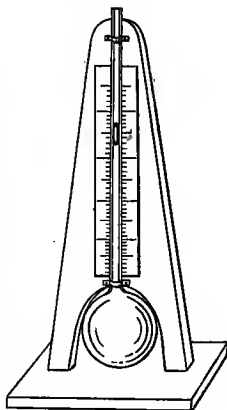


Fig. 7.

using such an air thermometer therefore it is necessary to know the pressure of the air, which is given by the height of the barometer, and to make an allowance if this pressure varies during an experiment.

An air thermometer is very much more sensitive than a mercury instrument for, for a given rise of temperature, a volume of air expands by about twenty times as much as the same volume of mercury. Thus if we had an air thermometer and a mercury thermometer with the same sized bulb and tube, the tube of the air thermometer would need to be about twenty times as long as that of the mercury thermometer in order that the two fixed points might be engraved on both. If however this were done and the two tubes divided each into 100 equal parts, as in the experiment with the sulphuric acid thermometer, we should find in this case that the two scales agreed throughout almost exactly. If the two be put into a bath in which the mercury thermometer read 15° , the air thermometer reads 15° also. The scale of the mercury thermometer is practically the same as that of the air thermometer. This will be seen to be of great importance, and we shall recur to the point again when discussing problems connected with the expansion of gases.

The form of air thermometer just described was used by Boyle in his experiments "On Cold" which were made about 1665.

Another form used by Boyle in the same experiments and called by him the open weather-glass is shewn in fig. 8.

A glass tube *A* passes through an air-tight cork into a bottle *B*, reaching nearly to the bottom of the bottle. The lower part of the bottle and part of the tube are filled with some coloured liquid. Above the liquid is the air or other gas, the expansion of which measures the temperature. As the temperature rises the pressure of the enclosed air increases and the liquid rises in the tube.

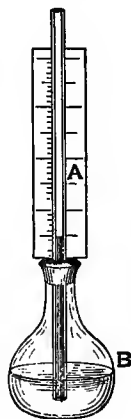


Fig. 8.

A third form of air thermometer was that used originally

in 1597 by Galileo, the inventor of the thermometer. This is shewn in fig. 9.

The air is contained in a bulb from the bottom of which a tube descends into an open beaker or bottle containing coloured liquid. By heating the bulb some of the air is forced out through the liquid. As the air cools the liquid rises in the tube, becoming stationary when the temperature has become constant. If the temperature changes slightly, the liquid moves. These instruments are all useful as delicate thermoscopes, they enable us to *detect* slight changes of temperature; they are difficult to use as thermometers or temperature measurers.

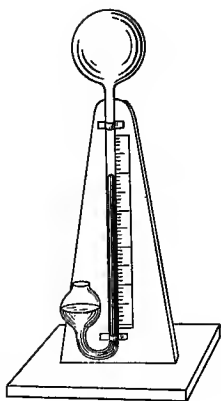


Fig. 9.

24. The differential Air Thermometer. In one of its forms this instrument consists of two bulbs with a tube joining them, bent as *ABCDE* in fig. 10. The portion *BCD* contains coloured liquid which, so long as the two bulbs are at the same temperature, stands at the same level within the tubes. If the temperature of one of them, *A* say, rises more than that in *E*, the pressure in *A* is increased, the end *B* of the liquid is depressed while *D* rises. Thus small differences of temperature between *A* and *E* are easily detected.

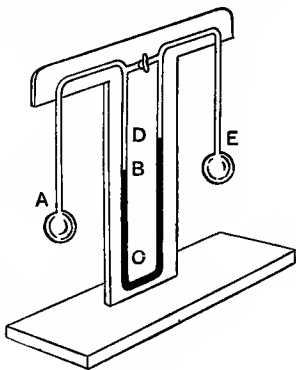


Fig. 10.

Reasons for the use of mercury in thermometers.

(1) The scale of a mercury thermometer agrees very closely with that of the air thermometer.

(2) Mercury readily transmits heat through its substance, so that the whole of the mercury in the thermometer very rapidly comes to the same temperature.

(3) It requires less heat to raise the temperature of a given mass of mercury than is required for an equal mass of most other liquids. This is of great importance, for when a thermometer is used to measure the temperature of a hot body it absorbs heat from that body until the temperatures of the two become equal. By this process the temperature of the hot body is reduced; if it requires much heat to raise the temperature of the thermometer the hot body might be appreciably cooled in the process and its temperature as measured by the thermometer might differ appreciably from its temperature before it was brought into contact with the thermometer.

(4) Other advantages are: mercury can readily be obtained pure; it does not wet glass and therefore does not stick so much to the tube. It is opaque and can be easily seen; it remains liquid over a considerable range of temperature, viz. from about -40°C. to 350°C.

25. Special forms of Thermometers. Rutherford's Maximum and Minimum Thermometers.

Rutherford's maximum thermometer is an ordinary mercurial thermometer usually mounted with the tube nearly horizontal. A small glass or enamel index (AB , fig. 11) in shape like a pin

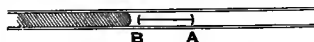


Fig. 11.

with a head at each end fits loosely into the tube above the mercury. As the temperature rises the convex surface of the mercury pushes the index forwards. As the temperature falls the mercury column retreats, leaving the index behind. The end B of the index indicates the highest temperature which the instrument has reached.

In Rutherford's minimum thermometer the same principle is applied. Alcohol is employed and the index is in the liquid. The surface is concave and as the column contracts it draws down the index, as it expands the alcohol passes by the index leaving it in the lowest position it has reached. Thus the end A of the index registers the minimum temperature to which the instrument has been exposed. Both instruments



Fig. 12.

are set by bringing the indices in contact with the end of the column of mercury or alcohol respectively.

26. The Clinical Thermometer. This is a sensitive maximum thermometer having only a short range, being usually graduated from about 95° Fah. to 110° Fah. A constriction is formed in the tube just above the bulb (fig. 13). As the

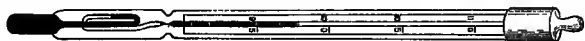


Fig. 13.

mercury rises it passes this constriction; as the temperature falls the thread breaks at the narrow part of the tube, the lower portion contracting into the bulb while the upper portion remains unchanged; in position the upper end of this portion registers the highest temperature to which the thermometer has been subject. By gently tapping or shaking the thermometer with the bulb downwards, the mercury can be forced past the constriction and the instrument set.

27. Six's Thermometer. This is a maximum and minimum instrument combined. The tube is bent as shewn in fig. 14. The bulb *A* and the tube *AC* are filled with alcohol, the expansion of which measures the temperature. The tube *BD* also contains alcohol, but the bulb *B* is only partly filled with it. The lower portion of the tube *CED* is filled with mercury separating the two columns of alcohol. The alcohol in *BD* merely serves the purpose of maintaining both ends of the mercury column under the same conditions. A small iron index is placed at each end of the mercury column, and a small spring attached to each index presses it against the side of the tube; the friction is sufficient to hold the index supported in any position. As the temperature rises, the alcohol in *A* expands; it pushes the end *C* of the mercury down and the end *D* up, raising the index at *D*. When the temperature falls again, the column at *D* falls and the index remains behind; the lower end of the index at *D* measures the maximum temperature. As

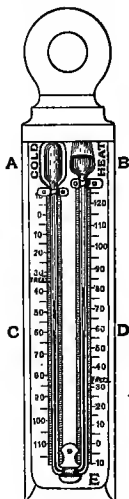


Fig. 14.

the temperature falls C rises, carrying with it the index, which in its turn remains suspended at the highest point it reaches. The lower end of this index then registers the minimum temperature.

The temperature is read on two vertical scales attached to the two tubes.

The instrument is set by bringing the indices into contact with the ends of the mercury column. This is done by the aid of a magnet.

EXAMPLES.

TEMPERATURE AND THERMOMETRY.

1. How is temperature measured by a mercury thermometer, and how are the fixed points of such a thermometer determined? Why is mercury selected for use in a thermometer?

2. What is an air thermometer? How is it constructed, and how is it used? What means have we, besides the air thermometer, of measuring temperatures between 400° and 800° C.?

3. Describe experiments illustrating the difference between temperature and heat.

4. A thermometer is graduated so that it reads 15 in melting ice and 60 in normal steam; convert into Centigrade degrees the readings 20 and 90 taken on that thermometer.

5. Describe some form of maximum and of minimum thermometer, explaining how they act.

6. Calculate the temperatures Centigrade corresponding to 100° F., -40° F., 0° F., 98° F.

7. Distinguish clearly between heat and temperature. Describe some form of a maximum and minimum thermometer.

8. Temperature may be defined as "The quality of a body in virtue of which it seems hot or cold;" and qualities are not capable of being *directly* measured: explain then the principle on which a thermometer is ordinarily used to measure temperatures.

Is it strictly correct to say that the temperature of one body is twice as great as that of another?

9. Convert the following readings of a thermometer graduated according to the Fahrenheit scale into degrees of the Centigrade scale: 86° , 0° , -22° ; also the following readings of a Centigrade thermometer into degrees of the Fahrenheit scale: 100° , -10° , -30° . What is the temperature at which the reading of a Fahrenheit thermometer is a number twice as large as that observed simultaneously on a Centigrade thermometer?

10. Describe the mode of determining the fixed points on a thermometer, and explain the nature of the correction necessary when the barometer stands higher than the normal height.

11. The latent heat of fusion of ice on the Centigrade scale is 80: find its value on the Fahrenheit scale.

12. How would you test the readings of an ordinary clinical thermometer?

CHAPTER IV.

CALORIMETRY. THE MEASUREMENT OF A QUANTITY OF HEAT.

28. Heat a Quantity. Heat is a Physical Quantity; if it requires a definite amount of heat to raise the temperature of a kilogramme of water from 15° to 16° , it will require the same quantity to raise a second kilogramme through the same range of temperature. If the two kilogrammes be mixed they require twice the amount of heat to raise their temperature which was required by each kilogramme separately; we can add together the two amounts of heat just as we can add together the two kilogrammes of water; we are justified in speaking of an amount or quantity of heat.

29. The Unit Quantity of Heat. We must measure a quantity of heat by some one of the effects it produces. In practice the effect chosen is the change it causes in the temperature of a definite mass of water. We state then as a **Definition**, *The unit quantity of heat is the amount of heat required to raise the temperature of 1 gramme of water from 4° C. to 5° C.*¹

To raise 2 grammes through this range will require 2 units of heat, and so on, so that m heat units is the quantity of heat required to raise m grammes of water from 4° to 5° .

It does not necessarily follow that 1 heat unit will raise a gramme of water 1 degree at any other part of the scale, say

¹ The reason for selecting this temperature will appear afterwards.

from 20° to 21° . Experiment shews however that this is very nearly the case, and so for most purposes we may take as a Heat Unit the quantity of heat required to raise 1 gramme of water 1° C. This same amount of heat is given out by 1 gramme of water in cooling 1° C.

30. EXPERIMENT (2). *To shew that the quantity of heat required to raise the temperature of a given mass of water 1° is very approximately the same at all parts of the scale.*

For this and many of the following experiments some kind of calorimeter is required.

A cylindrical vessel, fig. 15, made of thin copper about 10 cm. in diameter and 10 cm. high will serve the purpose. It may be mounted on three cork feet to prevent loss or gain of heat from direct contact with the table on which it rests. If greater accuracy is needed it should be suspended inside a second larger copper vessel. This larger vessel may be enclosed in a wooden box and protected

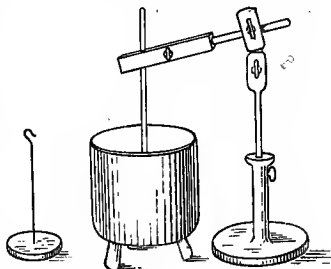


Fig. 15.

from external changes of temperature by cotton wool or other non-conducting packing. A definite quantity of water is placed in this calorimeter and quantities of heat are measured by the changes which take place in the temperature of this water. For accurate work and indeed for some of the experiments described below the water must be *weighed*; for most of the experiments it will be sufficient to *measure* it, pouring it into the calorimeter out of a measuring flask or a burette.

Measure out into the copper vessel a quantity, about 250 cubic centimetres, of warm water, at a temperature of about 30° C., and into a second vessel an equal quantity of water at about the temperature of the room, say 15° C. Note the temperatures, and immediately after having read the temperature of the warm water pour the cold into it and take the temperature of the mixture, it will be found to be approxi-

mately the mean of the two; thus in the given example it would be about $22^{\circ}5$. The heat given out by the 250 grammes of warm water in cooling through $7^{\circ}5$, from 30° to $22^{\circ}5$, is just sufficient to raise the temperature of the same mass of cold water through the same amount, viz. from 15° to $22^{\circ}5$. Thus it requires the same amount of heat to raise 250 grammes of water through $7^{\circ}5$, whether the rise be from 15° to $22^{\circ}5$ or from $22^{\circ}5$ to 30° .

As we have already stated, the law is not absolutely true, and if the experiments were done with great care, using precautions to prevent loss or gain of heat from external causes, the resulting temperature would not be the mean of the two initial temperatures, but rather higher than the mean.

31. Measurement of a Quantity of Heat. Since then the amounts of heat required to raise a gramme of water through each degree may for our purposes be considered as the same, it follows that $T - t$ heat units are needed to raise one gramme of water from t° to T° , i.e. through $(T - t)$ degrees.

Again the number of heat units required to raise m grammes of water 1° is m .

Hence the number of heat units required to raise m grammes of water from t° to T° is $m(T - t)$.

We thus see how to calculate either the amount of heat required to change the temperature of a given mass of water by a given number of degrees or the change in temperature produced by a given number of heat units.

Example. *Two quantities of water at different temperatures are mixed together, to find the temperature of the mixture.*

Let the masses of the two quantities be m_1, m_2 grammes and their temperatures t_1°, t_2° respectively.

Let the temperature of the mixture be t° . Then the quantity of heat absorbed by the mass m_1 in rising from t_1° to t° is $m_1(t - t_1)$ heat units, while the quantity given out by m_2 in cooling from t_2° to t° is $m_2(t_2 - t)$ heat units; if the experiment could be conducted so that there was neither loss nor gain of heat from other sources, these two quantities would be equal, the heat given by the hot body passes into the cold body and

$$m_1(t - t_1) = m_2(t_2 - t).$$

Thus

$$t = \frac{m_1 t_1 + m_2 t_2}{m_1 + m_2}.$$

This result may be verified by an experiment such as is described in § 30.

32. Specific Heat. The unit quantity of heat raises the temperature of 1 gramme of water 1°C . Experiment shews that this same quantity of heat will raise the temperature of a gramme of most other substances more than 1°C . Thus a gramme of Lead or Mercury would be raised about 30°C . by one heat unit, a gramme of Silver or Tin about 20°C ., a gramme of Copper 10° . Conversely equal masses of different bodies in cooling through the same range of temperature give out different quantities of heat.

This is shewn by the following experiment. A number of balls of different materials, lead, tin, copper, zinc, iron, etc. and of about the same mass are placed in a vessel of boiling oil or water. By means of a string or fine wire attached to each ball it can be readily removed from the bath. Remove the balls simultaneously and place them on a cake of paraffin or beeswax supported on the ring of a retort stand. This cake is prepared by melting the paraffin and allowing it to run out into a flat circular vessel. The balls will melt the paraffin, but the amount melted by each ball will be different.

If the wax be not too thick the iron, zinc and copper balls may melt through it, but the time taken in so doing will be different, the tin ball will not penetrate so deeply while the lead ball will melt less wax than any of the others. It is clear then that the different balls give out different quantities of heat; the experiment however could not be used to measure the amount of heat given out, for much of the heat will escape into the air, and the rate at which the melting takes place depends on the rate at which heat can pass through the substance of the balls as well as on the amount of heat given out by the balls.

The following experiment however will give more measurable results.

33. EXPERIMENT (3). *To compare the amounts of heat given out by different bodies in cooling through the same range of temperature.*

Take a number of bodies, lead, tin, copper, etc. having small wire handles attached, of equal mass, M grammes; M should be about 400 grammes; and place them in a vessel of boiling water.

Measure out into the copper calorimeter (Exp. 2) m cubic centimetres of water; the mass of this is m grammes— m may conveniently be about 500. Take its temperature with a sensitive thermometer. The thermometer should read to fifths of a degree Centigrade—let it be t° . Now take one of the bodies, say the copper, out of the bath of boiling water and place it rapidly in the calorimeter; in doing this care must be taken to carry as little of the hot water as possible with the copper. Observe the temperature of the water, keeping it stirred by moving the copper about; the temperature rises for a time and then becomes stationary. Note this temperature, let it be T° . Then, if for the present we suppose that no heat was lost in transferring the copper to the calorimeter, and that all the heat given out by the copper has passed into the water, since the copper has fallen in temperature from 100° to T° and the water has risen from t° to T° , we find that M grammes of copper in falling through $(100 - T)$ degrees can raise m grammes of water through $(T - t)$ degrees; that is the copper gives out $m(T - t)$ heat units.

Thus M grammes of copper in falling 1° give out

$$m(T - t)/(100 - T) \text{ heat units.}$$

Repeat the experiment with the lead and tin, the resulting temperature T in each case will be different. Thus the quantities of heat given out by equal masses of copper, tin, and lead in cooling 1°C. , are different. With the numbers given if t the original temperature be 15° the values of the final temperature for copper, tin and lead respectively, will be about 21.1 , 18.7 and 17.2 .

Thus while the copper raises the temperature of the water 6.1 , the tin only raises it 3.7 and the lead 2.2 . The fall of temperature of the copper moreover is less than that of the tin or lead; thus we see that the amount of heat given out by 1 gramme of copper in cooling 1° is greater than that given out by 1 gramme of tin or lead under similar circumstances.

34. Definition of Specific Heat. *The ratio of the quantity of heat required to raise the temperature of a given mass of any substance 1° to the quantity of heat required to raise the temperature of an equal mass of water 1° is called the specific heat of the substance.*

35. Definition of Capacity for Heat of a Body. *The Capacity for heat of a body is the quantity of heat required to raise the temperature of the body 1° .*

36. Relation between Specific Heat and Capacity for Heat. Let there be m grammes of the substance and let C be its specific heat. Then by the definition

$$C = \frac{\text{No. of units of heat required to raise } m \text{ grms. of substance } 1^\circ}{\text{No. of units of heat required to raise } m \text{ grms. of water } 1^\circ}$$

But (§ 31) the number of units of heat required to raise m grammes of water 1° C. is m .

$$C = \frac{\text{No. of units of heat required to raise } m \text{ grms. of substance } 1^\circ}{m}$$

Therefore the number of units of heat required to raise m grammes of a substance $1^\circ = mC$. But this quantity is the capacity for heat of the body.

Thus mC is the capacity for heat of a body containing m grammes of a substance of specific heat C .

Again mC heat units will raise mC grammes of water 1° . Hence mC is also the number of grammes of water which will be raised 1° by the heat required to raise the body 1° . This mass of water is called the water equivalent of the body.

37. Definition of water equivalent of a body. *The water equivalent of a body is the number of grammes of water which will be raised 1° by the heat required to raise the temperature of the body 1° .*

Thus in the units we have taken, the capacity for heat of a body and its water equivalent are numerically the same.

38. EXPERIMENT (4). *To find by the method of mixture the specific heat of a given substance.*

Proceed as in Experiment (3) and from the data there given calculate the specific heat thus. Let C be the specific heat of the substance, M its mass, T the final temperature; let m be the mass of the water, t its initial temperature. The substance has fallen in temperature $(100 - T)$ degrees; the heat emitted in falling 1° is MC .

Hence the heat emitted is

$$MC (100 - T) \text{ units of heat}$$

The heat absorbed by the water is

$$m (T - t) \text{ units,}$$

for its mass is m grammes and it has risen through $(T - t)$ degrees.

If for the present we assume that all the heat emitted by the substance goes into the water we have

$$MC (100 - T) = m (T - t),$$

$$\therefore C = \frac{m (T - t)}{M (100 - T)}.$$

If we take the numbers found in Experiment (3) for copper, we have

$$m = 500, \quad M = 400, \quad T = 21^\circ.1, \quad t = 15^\circ,$$

and from this we have

$$C = \frac{500}{400} \cdot \frac{6.1}{78.9} = .093.$$

39. The following table gives the values of the specific heat of a few substances.

Copper	.0951	Mercury	.0333
Zinc	.0955	Carbon disulphide	.221
Iron	.1138	Turpentine	.467
Lead	.0315	Glass	.1877
Gold	.0324	Ice	.5
Platinum	.0324	Ether	.517
Tin	.0562	Alcohol	.615
Silver	.0570	Paraffin	.683

40. Defects of the experiment.

Several sources of error affect the experiment as just described.

(1) Heat is used in raising the temperature of the calorimeter and thermometer as well as that of the water; hence the heat emitted by the hot body does not all pass into the water, some goes into the calorimeter. In consequence the specific heat found will be too low.

(2) The hot body cools very rapidly in being transferred from the bath of boiling water to the calorimeter; its temperature will be less than 100° when it is dropped into the water. This again will make the observed specific heat too low.

(3) Heat is lost by radiation or conduction from the calorimeter to surrounding bodies assuming the calorimeter at a higher temperature than the room, or gained by the calorimeter from those bodies if the calorimeter be cooler than they are.

(4) In transferring the hot body from the bath to the calorimeter some hot water is transferred also. This will make the observed value too high.

(5) The temperature of the boiling water is probably not accurately 100°C .

The errors which would arise from these are corrected partly by calculation, partly by modifying the arrangement of the apparatus.

(1) We can allow for this if we know the amount of heat required to raise the calorimeter, stirrer and thermometer 1° . Let this be m_1 heat units. Now the calorimeter etc. are initially and finally at the same temperature as the water. They are raised therefore through $T - t$ degrees, and the heat required for this will be $m_1(T - t)$.

Thus the total heat absorbed is

$$m(T - t) + m_1(T - t)$$

and we have

$$\begin{aligned} MC(100 - T) &= m(T - t) + m_1(T - t) \\ &= (m + m_1)(T - t). \end{aligned}$$

Thus the correction shews itself as an addition to the mass of water in the calorimeter; this is increased by the water equivalent of the calorimeter, stirrer etc. This correction which explains the name "water equivalent" is what we should expect. So far as our problem is concerned we may clearly suppose that the calorimeter absorbs no heat, but that it contains such an additional quantity of water as requires, to produce a rise of temperature of 1° , the same amount of heat as is required by the calorimeter. This quantity of water we have defined to be the water equivalent of the calorimeter.

We can determine the water equivalent of the calorimeter experimentally; in practice it is best found from the fact that it is numerically equal to the capacity for heat of the calorimeter. Thus if m_c be the mass of the calorimeter, c its specific heat, we have for its water equivalent

the value $m_c \times c$. The water equivalent of the thermometer is included in the term m_1 but it is very small. In the apparatus described the value of m_c may be about 100 grammes, the value of c for copper is approximately $\cdot 1$, so that the value of $m_c \times c$ is about 10. The calorimeter for this purpose is equivalent to 10 grammes of water. We ought therefore to suppose that the calorimeter contains $500 + 10$ grammes instead of the 500, and the value of the specific heat will be about $\cdot 095$.

(2) and (4). These are guarded against by a suitable arrangement of apparatus. This as usually employed consists of a steam heater, an outer cylinder of thin copper *A*, fig. 16, closed at both ends but with

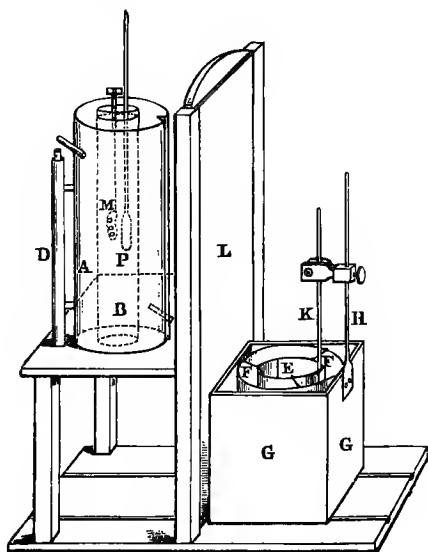


Fig. 16.

a tube *B* passing through it. The body to be heated hangs by a thread in this inner tube. A thermometer *P* gives its temperature. The upper end of *B* is closed by a cork through which the thread and thermometer pass. Steam can be passed through the outer cylinder *A* from a boiler to a condenser, thus heating *M* without bringing it in contact either with the steam or the water.

The cylinder turns round a vertical axis *D* above a horizontal board. A circular hole is cut in the board, which in one position of the cylinder

comes directly under the tube B , for other positions of the cylinder the end of the tube is closed by the board. The calorimeter E is contained in a wooden box G to the bottom of which slides are fixed. These run in grooves in the base of the apparatus, and the box can be readily shot under the board to which the heater is fixed. When this is done, the calorimeter is directly under the hole in this board, and when the heater is turned so that the tube B may come vertically over this hole the substance M can be dropped gently into the calorimeter, which is then withdrawn and the rise of temperature measured. To prevent heat reaching the calorimeter by radiation from the heater, a board L sliding in vertical grooves is interposed. When the calorimeter is to be placed beneath the heater this board is raised, when the calorimeter is withdrawn it is again lowered. The hot body is thus raised to a known temperature without becoming wet and can be dropped into the water of the calorimeter without much loss of heat. Its exact temperature is given by the thermometer and thus the error referred to in (5) is avoided.

(3) To avoid loss or gain of heat from external causes, the calorimeter E is polished brightly on the outside (see § 167), and suspended by three strings within a second copper vessel polished on the inside; this second vessel is placed in the wooden box with a packing of wool or felt between it and the box. There is also a lid to the box not shewn in the figure, with a hole through which the thermometer passes. The lid is removed while the hot body is being placed in the calorimeter. (Glazebrook and Shaw, *Practical Physics*, p. 274.)

41. Latent Heat. Heat is required to melt a solid. Thus ice melts at a temperature of 0°C . if heat be applied to it. Lead melts at 327°C . approximately, sulphur at 113°C ., silver at 1000°C .

Moreover we can shew that a definite quantity of heat is required to melt a given mass—say 1 gramme—of each of these substances, and further that the temperature of the mass remains constant while the melting is in progress.

Definition of Latent Heat of Fusion. *The quantity of heat required to change 1 gramme of a substance from a solid to a liquid form without change of temperature is called the latent heat of fusion of the substance.*

Hence if L be the latent heat of fusion of any substance, the quantity of heat required to melt m grammes of that substance without changing its temperature is mL heat units. Now suppose that m grammes of ice are put into a calorimeter containing M grammes of water at a temperature T° . The ice, if there be not too much of it, melts and the temperature of the water falls. Let it fall to t° . The heat emitted by the

M grammes of water as it cools from T° to t° is used in melting the m grammes of ice and in raising the temperature of the m grammes of water formed from 0° to t° ¹.

The heat required for this is $Lm + mt$ units. The heat emitted by the water is $M(T - t)$: if we omit the small corrections these two quantities of heat are to be taken as equal and thus

$$Lm + mt = M(T - t),$$

$$\therefore L = \frac{M}{m}(T - t) - t.$$

Now experiment shews that we may vary the mass of ice or of water or the initial temperature of the water and still always get the same value for L . This value is about 79.2 heat units. We are therefore justified in saying that the amount of heat required to melt 1 gramme of ice is constant and in speaking of "the latent heat of fusion of ice."

42. EXPERIMENT (5). *To find the latent heat of fusion of ice.*

Weigh the calorimeter empty; pour water into it and weigh again. The difference between the two weights gives the mass M of the water².

Break up some ice into small pieces. Take the temperature of the water, let it be T° (T should be about 20°). Dry each piece of ice on some flannel or cloth and drop it into the water. A stirrer of wire gauze fits the inside of the calorimeter and by placing this above the ice it can be kept below the surface of the water; introduce thus about 50 grammes of ice³. As the ice melts the temperature falls; note the lowest

¹ If the temperatures be Fahrenheit, since freezing point is 32° Fahr., the melted ice is raised from 32° to t° ; hence in this case

$$Lm + m(t - 32) = M(T - t),$$

or

$$L = \frac{M}{m}(T - t) - (t - 32).$$

² It is necessary to weigh the calorimeter and water in this experiment and not merely to measure the water put in as in § 33, for the mass of ice introduced can only be obtained by weighing the calorimeter and its contents when the ice is melted; we must therefore know the mass of the calorimeter and water before introducing the ice.

³ Ice should be added until the fall in temperature is about 10° .

temperature, let it be t° . To find the mass of ice added weigh the calorimeter and its contents again. The increase in mass gives m the mass of ice added, and we thus determine experimentally all the quantities but L in the formula above.

Let us suppose we find

Mass of Calorimeter = 110 grammes.

Mass of Calorimeter and Water = 630 grammes.

Mass of Water = M = 520 grammes.

Mass of Calorimeter, Water and Melted Ice = 685 grammes.

Mass of Ice = m = 55 grammes.

Observed temperatures $T = 19^\circ\cdot8$, $t = 10^\circ\cdot5$.

Then from the formula $L = \frac{M(T-t)}{m} - t$ we obtain as the latent heat of fusion of ice $77\cdot5$.

43. Sources of error in this experiment.

(1) The heat given up by the calorimeter in cooling from T to t has been neglected. In consequence the value of L is too small.

(2) There may be a transference of heat between the calorimeter and surrounding bodies.

(3) Some water has been carried into the water with the ice. This again makes L too small.

To correct for (1) the water equivalent of the calorimeter must be known and added to M as described in § 40. The equivalent of the given calorimeter if of copper is about 11, and the number 520 will in consequence become 531.

(2) The correction for this is most easily effected by arranging that the temperature t shall be as much below the temperature of the room as T is above it. For half of the experiment the calorimeter is losing heat, for the other half it is gaining it, and the loss and gain about balance. If the temperature of the room be about 15°C . this is secured in the given experiment.

(3) The only way to avoid this is to dry each piece of ice with care before it is inserted. Its effect is clearly to make L too low, for the mass m as found by the experiment is too high, being that of the ice and the water which adhered to it.

44. Latent heat of Vaporization. Just as heat is required to change a given mass of solid into liquid without

change of temperature, so heat is required to change a given mass of liquid into vapour.

45. Definition of Latent heat of Vaporization.

The Latent heat of Vaporization of a liquid is the amount of heat required to convert 1 gramme of the liquid into vapour without change of temperature.

Thus if L be the latent heat of vaporization of water, at any given temperature, the quantity of heat required to convert m grammes of water into steam at that temperature is mL .

This quantity is also given out when m grammes of steam are condensed to water. Suppose now that m grammes of steam are passed into a calorimeter containing M grammes of water at $T^\circ\text{C}$. The temperature rises; let it become $t^\circ\text{C}$. and let the steam be at 100°C . Then m grammes of steam are condensed to water at 100° and then cooled from 100° to t° . Heat is given out in both these processes and the total amount thus evolved is

$$mL + m(100 - t).$$

This amount of heat raises M grammes of water from T to t° and the heat required for this is

$$M(t - T).$$

If as before we suppose that the heat emitted by the steam passes into the water, we have

$$mL + m(100 - t) = M(t - T),$$

$$\therefore L = \frac{M}{m}(t - T) - (100 - t).$$

If we perform the experiment varying the masses M and m and the temperatures T and t , we find that we obtain the same number for L .

This value is 536 heat units, and when we state that the latent heat of vaporization of water is 536 we mean that 536 heat units are required to convert 1 gramme of water at 100°C . into steam at 100°C . This is sometimes called the latent heat of steam.

46. EXPERIMENT (6). *To find the latent heat of vaporization of water.*

Weigh the calorimeter empty; pour into it about 500 c.c. of water, at the temperature of the room, say 15° , or if it can conveniently be obtained, at some lower temperature, say 5° , and weigh again. The difference gives the mass M of water in the calorimeter. Boil some water either in a metal boiler or in a glass flask. The mouth of the flask is closed with a cork through which a glass tube passes. To this about a foot of india-rubber tubing is attached, and at the end of the rubber tubing a glass nozzle is fitted. As the water boils the steam issues from the nozzle. The steam will usually carry with it a certain amount of condensed water. This may be reduced in quantity by wrapping wool or felt round the india-rubber tube; most of the water left may be caught by a suitable trap shewn in fig. 17. This consists of a wide glass tube closed with corks through each of which a glass tube passes. To one of these, passing through the top of the wide tube, the india-rubber tube is fitted; the other which passes through the bottom of the wide tube, but reaches inside it nearly to the top, is connected with the nozzle; the whole is covered with wool; the greater portion of the water conveyed by the steam is deposited in the lower part of the wide tube, the steam which issues from the nozzle is nearly dry. The calorimeter is placed in a convenient position near the boiler and protected from direct radiation from the boiler by a screen. When the steam is issuing freely from the nozzle, take the temperature of the water in the calorimeter, let it be T° . Bring the nozzle quickly below the surface of the water in the calorimeter, the issuing steam is condensed by the cold water and the temperature given by the thermometer rises somewhat rapidly; when it has risen about 20° withdraw the nozzle, taking care to carry off as little water as possible. Stir the water and note the highest temperature which the thermometer reaches, let it be t° ; weigh the calorimeter again, the increase in mass gives the mass m of steam condensed. Then we have sufficient data to find L .



Fig. 17.

Thus suppose we find

Mass of Calorimeter = 110 grammes.

Mass of Calorimeter and Water = 505 grammes.

Mass of Water = M = 495 grammes.

Mass of Calorimeter, Water and
Condensed Steam = 522 grammes.

$m = 17$ grammes.

$T = 15^{\circ}\cdot 2.$ $t = 35^{\circ}\cdot 4.$

Hence the condensed steam is cooled from 100° to $35^{\circ}\cdot 4$ or through $64^{\circ}\cdot 6$, and the heat given out by the 17 grammes is

$$17L + 17 \times 64\cdot 6.$$

This heat raised 495 grammes of water from $15^{\circ}\cdot 2$ to $35^{\circ}\cdot 4$ or through $20^{\circ}\cdot 2$. For this $495 \times 20\cdot 6$ units are needed ;

$$\therefore L + 64\cdot 6 = \frac{495 \times 20\cdot 2}{17} = 588\cdot 2,$$

$$\therefore L = 523\cdot 6.$$

47. Sources of Error. These are much the same as in the last experiment.

(1) Some of the heat given up by the steam has been used in raising the temperature of the calorimeter. In consequence the value of L found is too low.

(2) There has been loss of heat by radiation, from this cause also L is too low.

(3) Some hot water has been carried over, thus m is too big and therefore L is too low.

(1) To correct for this, the water equivalent of the calorimeter must be added to M .

(2) An estimate of the rate of cooling can be made by a suitable experiment (see § 170), and the error allowed for. Its magnitude would be reduced by starting at a low temperature, say 5° , and raising the water finally as much above the temperature of the room as it was initially below it.

(3) Care must be taken to dry the steam.

48. Ice Calorimeters. The facts that a definite quantity of heat is required to melt a given quantity of ice or

to vaporize a given quantity of water have both been used as a means of measuring quantities of heat.

Thus various forms of ice calorimeters have been devised.

In the simplest form as used by Black, fig. 18, a hole is cut or bored into a block of ice. This can be covered by a slab of ice. The hot body of known mass M is raised to a known temperature T and dropped into the cavity, the cover is put on and after a short time it is removed. The water formed from the melted ice is sucked up into a pipette and its mass determined by weighing or measuring its volume; let it be m grammes and let L be the latent heat of fusion of the ice, C the specific heat of the substance. Then M grammes in cooling through T degrees from T° to 0° give out MCT heat units, and this heat melts m grammes of ice for which mL heat units are required,

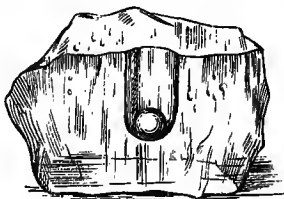


Fig. 18.

$$\therefore MCT = mL,$$

$$\text{and } C = \frac{mL}{MT}.$$

In this form however the measurement will not give accurate results, for (1) it is impossible to prevent some heat from getting into the cavity from sources other than the hot body and (2) it is impossible to get all the water out of the cavity and measure its mass accurately. These difficulties are to some extent overcome in Lavoisier and Laplace's Calorimeter, though by no means completely.

*49. Ice Calorimeter of Lavoisier and Laplace.

The instrument, fig. 19, consists of three copper vessels. The inner one A is to contain the hot body. It is fitted with a lid and placed inside a second copper vessel B , the space between the two is filled with ice broken into small pieces. A tube passes through the outermost vessel C into B . The vessel B also has a lid and is surrounded by the third vessel C , the

space between *B* and *C* is filled with ice and a tube passes into *C*. Thus the ice in *B* is entirely surrounded by the ice in *C*, and so heat from outside cannot pass into *B*. So long as *C* is fairly filled with ice, the heat from outside melts ice in *C* and the temperature remains at zero; no ice is melted in *B*. If the lids be now removed and a hot body be placed in *A*, the lids being quickly replaced, some of the ice in *B* is melted by the heat emitted by the hot body in cooling down to zero, and the water formed from the melting ice runs out through the tube into a vessel placed to receive it. The mass of water thus produced can be weighed. And the specific heat of the solid can be found from the same equation as in § 48 above. For if *M*, *T* and *C* be the mass, temperature and specific heat of the substance, *m* the mass of the water formed, we have $MCT = \text{heat given out by substance in cooling from } T^{\circ} \text{ to zero} = \text{heat used in melting } m \text{ grammes of ice} = mL$,

$$\therefore C = \frac{mL}{MT}.$$

Even in this form however the instrument is not accurate, for a considerable portion of the water formed in *B* sticks to the ice, thus *m* and therefore *C* are too small.

***50. Bunsen's Ice Calorimeter.** The action of this instrument depends on the fact that ice on melting diminishes in volume by a known amount, so that if the change of volume be measured the amount melted and hence the quantity of heat employed in melting the ice can be measured.

This calorimeter consists of a glass tube *A*, fig. 20, like a large test tube which is sealed inside a larger glass vessel *B*. At the bottom of this there is attached a small tube *CDE* bent as shewn in the figure. The bore of the horizontal portion of this should be fairly uniform and the area of its cross section should be known. This tube and the lower part

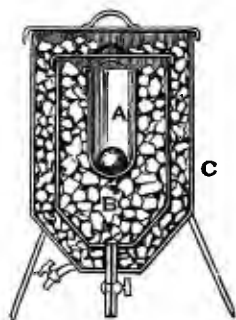


Fig. 19.

of the vessel *B* are filled with mercury. The upper part of *A* is filled with water from which the air has been removed by long-continued boiling.

To use the instrument it is packed in ice inside another vessel, leaving only the horizontal tube *DE* projecting; some of the water in *A* is then frozen.

This is done by passing through *B*, by means of a pump, alcohol which has been cooled below 0°C . by exposure to a freezing mixture. As the ice is formed the combined volume of the water and ice in *A* increases and some of the mercury is forced out of the tube *DE*. When the freezing has been sufficient the alcohol is removed and a quantity of water which has been cooled down to 0°C . is

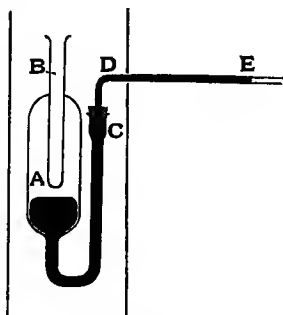


Fig. 20.

placed in *B*. If the calorimeter is completely packed in ice no heat can now reach the ice in *A* from outside and the combined volume of the water and ice in *B* remains constant, the end of the mercury column takes up a definite position in the tube *DE*,—a scale of millimetres is attached to this tube and thus the position of the end of the column can be noted. Let us suppose this steady state has been attained, and that the mercury in *DE* stands at a point P_1 .

Place a hot body in the water in *B*, let M , C , T be its mass, specific heat and temperature. The heat given up by this body in cooling to 0°C . melts some of the ice in *A*. The combined volume of ice and water is reduced and the mercury recedes in the tube, becoming steady again finally at a second point P_2 ; when this is reached the body has been cooled down to 0° and all the heat given up by it has been employed in melting the ice¹. Measure the distance P_1P_2 in cm., and by

¹ Very little heat escapes from the top of the water in *B*, for as the temperature of the water begins to rise above 0° its density increases slightly (see § 90) and the warmed water remains at the bottom and melts the ice.

multiplying it by the area of the cross-section of the tube in square cm., find the change in the combined volume of water and ice in A , let it be v c.c. Now the volume of 1 gramme of ice 0° is 1.091 c.c., while that of 1 gramme of water is 1 c.c. Thus when 1 gramme of ice becomes water there will be a diminution of volume of .091 c.c., so that to produce contraction of v c.c. the number of grammes of ice melted will be

$v/.091$, and this comes to be $v \times 10.99$ grammes.

To melt this $L \times v \times 10.99$ heat units are required.

Hence $MCT' = L \times v \times 10.99$,

and
$$C = \frac{10.99 Lv}{MT}.$$

***51. The Steam Calorimeter.** In the steam calorimeter recently devised by Mr Joly, a body of mass M whose specific heat is to be found is suspended by a fine platinum wire inside a calorimeter. The wire passes through a small hole in the calorimeter and is suspended from one arm of a delicate balance. The body is weighed and its temperature is observed. Let it be T° .

Steam is then let into the calorimeter. Some of the steam is condensed on the body, raising its temperature until it reaches 100° . The mass of steam so condensed can be found by again weighing the body in the steam; let this mass be m grammes. Then m grammes of steam in condensing to water at 100° C. have raised the temperature of M grammes of the body from T° to 100° . Hence if C be the specific heat of the body, L the latent heat of steam,

$$MC(100 - T) = mL,$$

$$C = \frac{mL}{M(100 - T)}.$$

***52. Other forms of Calorimeters.** Various other forms of calorimeters have been devised for special purposes. Thus in Regnault's experiments on the specific heat of gases he employed two long coils of thin copper tubing through which the gas was passed; there were arrangements for measuring the amount of gas which passed through and for

regulating its pressure which was kept constant. One of these coils was immersed in a vessel of boiling oil at a known high temperature, the other was in the water of the calorimeter. As the gas passed slowly through, it acquired the temperature of the oil bath in the first coil, in traversing the second coil it parted with its heat to the water in the calorimeter which in consequence rose in temperature. From this rise in temperature, combined with a knowledge of the mass of water in the calorimeter, the water equivalent of the coil and calorimeter and the temperature of the oil bath, the specific heat of the gas can be found.

The specific heat of a liquid can be found by the method of mixture in various ways. One is to enclose the liquid in a thin copper vessel, to heat this in the same manner as a solid either in hot water or in the steam heater and then immerse it in the water in the calorimeter. In making calculations the heat contributed by the copper vessel must be taken into account.

The following **Example** will shew how this is done. 250 grammes of turpentine enclosed in a copper vessel whose mass is 25 grammes are heated to 100°C . and immersed in 589 grammes of water at 13°C . in a copper calorimeter of which the mass is 110 grammes. The temperature rises to $27^{\circ}\cdot 5$. Assuming the specific heat of copper to be $\cdot 1$ find that of turpentine.

Let C be the specific heat of turpentine.

Then heat given out by turpentine in falling from 100° to $27^{\circ}\cdot 5$
 $= 250 \times C \times 72\cdot 5$.

Heat given out by copper vessel in falling from 100° to $27^{\circ}\cdot 5$
 $= 25 \times \cdot 1 \times 72\cdot 5$.

\therefore Total heat given out $= (250C + 2\cdot 5)72\cdot 5$.

Heat absorbed by water rising from 13° to $27^{\circ}\cdot 5$
 $= 589 \times 14\cdot 5$.

Heat absorbed by calorimeter in rising from 13° to $27^{\circ}\cdot 5$
 $= 110 \times \cdot 1 \times 14\cdot 5$.

\therefore Total heat absorbed $= (589 + 11) 14\cdot 5$.

$\therefore (250C + 2\cdot 5) 72\cdot 5 = 600 \times 14\cdot 5$.

And from this we find $C = \cdot 470$.

Or again, we may determine the specific heat of a liquid by raising some solid of known specific heat to a given temperature, immersing it in a known mass of the liquid and measuring the rise of temperature.

Thus 50 grammes of glass are heated to 100° and immersed in 250 grammes of alcohol at 15° . The temperature rises to 20° . The specific heat of glass is $\cdot 198$, find that of alcohol.

Let c = specific heat of alcohol.

The heat given out by the glass in cooling from 100° to 20°

$$= 50 \times \cdot 198 \times 80.$$

The heat absorbed by the alcohol in rising from 15° to 20°

$$= 250 \times c \times 5.$$

$$\therefore c \times 250 \times 5 = 50 \times 80 \times \cdot 198.$$

Whence

$$c = \cdot 63.$$

Another form of Calorimeter is Favre and Silbermann's. This consists of a large iron sphere which is filled with mercury; a glass tube of narrow bore is fitted to this, and the mercury is visible in the tube. The instrument is thus like a large thermometer. A tube of thin iron in shape like a test tube penetrates to the centre of the sphere. If a hot body is placed in this tube it communicates its heat to the mercury through the thin iron; the mercury rises in the glass tube. The whole apparatus is packed in some non-conducting material so that all the heat communicated to the mercury is retained by it and the expansion of the mercury is thus proportional to the amount of heat communicated to it. If then we know the temperature of the hot body when it was inserted in the tube and the temperature to which it falls, we can determine the amount of heat it has given out in falling through a known range of temperature and thus find its specific heat.

EXAMPLES.

CALORIMETRY. (SPECIFIC HEAT AND LATENT HEAT.)

1. Describe any good method of determining the specific heat of a solid substance.

The latent heat of tin is $14\cdot 25$, its specific heat $\cdot 064$, and its melting point 232° on the Centigrade scale. What will be the corresponding numbers on the Fahrenheit scale?

2. What is meant by a *unit of heat*? Taking the specific heat of lead as $\cdot 031$, and its latent heat as $5\cdot 07$, find the amount of heat necessary to raise 15 lbs. of lead from a temperature of 115°C. to its melting point, 320°C. , and to melt it.

3. Define the boiling point and the latent heat of vaporization of a liquid. Why is it necessary to note the height of the barometer when determining the upper fixed point of a thermometer? How is this determination made? An alteration of pressure of 26 mm. alters the boiling point of water 1° . Find the boiling point when the height of the barometer is 745 mm.

4. Define latent heat, specific heat, capacity for heat. The specific heat of copper is $\cdot 095$; calculate the capacity for heat of a kilogramme of copper. A kilogramme of copper is raised to a temperature of 100°C . and placed in an ice calorimeter. How much ice will be melted assuming the latent heat of fusion of ice to be 80?

5. What do you understand by the latent heat of fusion and the latent heat of evaporation of a substance? Explain a method for determining the latent heat of evaporation of water.

6. Explain the statement that the latent heat of water is 80. To a pound of ice at 0° are communicated 100 units of heat (pound-degrees Centigrade). What change of temperature does the ice undergo, and in what way is its volume altered?

7. $2\frac{1}{2}$ kilogrammes of iron (specific heat $\cdot 112$) at 95°C . are put into 3 litres of water at 15°C . Find the rise of temperature of the water.

8. A mass of 200 grammes of copper, whose specific heat is $\cdot 095$, is heated to 100°C . and placed in 100 grammes of alcohol at 8°C ., contained in a copper calorimeter, whose mass is 25 grammes: the temperature rises to $28^{\circ}\cdot 5\text{C}$. Find the specific heat of alcohol.

9. If 25 grammes of steam at 100°C . be passed into 300 grammes of ice-cold water, what will be the temperature of the mixture, the latent heat of steam being taken equal to 536?

10. In 100 grammes of boiling water ($t=100$) there are placed 20 grammes of ice, and the temperature falls to 70° when the ice is just melted. Determine the latent heat of fusion of ice, assuming no heat to be lost.

11. What is meant by the statement that the "latent heat of steam is 536"?

Steam is passed into 100 grammes of water at 15° in a calorimeter. If the mass of water in the calorimeter be by this means increased by 10 grammes, find the final temperature, supposing no heat to have been lost and that the heat absorbed by the calorimeter may be neglected.

12. Five grammes of a substance at 100°C . are placed in a Bunsen's Calorimeter and the volume of the ice and water is found to decrease by $\cdot 05$ cubic centimetres. Assuming that water expands by one eleventh of its volume on freezing and that the latent heat of fusion is 79, find the specific heat of the substance.

13. What is the water equivalent of a vessel, and how may it be determined?

14. A glass beaker contains 1 lb. of water and some ice. When 1 oz. of steam at 100°C . has been supplied to the vessel the contents are at a temperature of 3°C .; how much ice was there to begin with?

(Latent heat of ice = 79, of steam = 537.)

15. Find the specific heat of a substance of which 375 grammes at 100°C ., when immersed in 280 grammes of water at 15°C ., raise the temperature of the water to 25°C .

16. Into a mass of water at 0°C . 100 grammes of ice at -12°C . are introduced, 7.2 grammes of the water are frozen and the temperature of the ice rises to 0°C .; if the specific heat of ice be .5, find its latent heat of fusion.

17. Three liquids *A*, *B*, *C*, are given. Four grammes of *A* at 60° , and one gramme of *C* at 50° , have, after mixing, a temperature of 55° . A mixture of one gramme of *A* at 60° and one gramme of *B* at 50° shews a temperature of 55° . What would be the temperature of a mixture of one gramme of *B* at 60° , and one gramme of *C* at 50° ? Would the result be different according as the thermometer readings given are those of the Fahrenheit or the Centigrade scale?

18. What are the "latent heat" and the "total heat" of steam? Describe the effect of compressing saturated steam and of allowing it to expand "without loss or gain of heat."

19. Explain why in a Bunsen's Calorimeter there is no need to determine the temperature of the liquid kept in the test tube of the calorimeter when the capacity for heat of a solid is determined.

20. A quantity of turpentine, 250 grammes in mass, is enclosed in a copper vessel whose mass is 25 grammes and is heated to 100°C . On immersing the whole in 535 grammes of water at 13°C . in a copper calorimeter 110 grammes in mass the temperature rises to 27.5°C . Assuming the specific heat of copper to be .1 find that of turpentine.

21. What will be the result of placing (a) 5 lbs. of copper at 100°C ., (b) 30 lbs. of copper at 80°C ., in contact with $1\frac{1}{2}$ lbs. of ice at 0°C .?

(Specific heat of copper = .092, latent heat of fusion of ice = 79.)

22. A ball of copper (Sp. heat = 0.092), whose mass is 5 lbs. is heated in a furnace, and allowed to drop into a gallon of water at 10°C .; the temperature of the water rises to 50°C .; find the temperature of the furnace. What are the objections to this method of measuring high temperatures?

23. A calorimeter of copper weighs 80 grammes and the specific heat of the copper is .092. It contains 100 grammes of water at 10°C . Steam at 100°C . is passed into it until the temperature rises to 80°C . when the total weight of water is found to be 113.42 grammes. What value does this give for the Latent Heat of Steam?

24. The specific heat of copper is $\cdot 095$; calculate the capacity for heat of a kilogramme of copper. A kilogramme of copper is raised to a temperature of 100°C . and placed in an ice calorimeter. If $1\cdot 18$ grammes of ice are melted find the latent heat of fusion of ice.

25. 20 grammes of a substance, at a temperature of 10°C ., whose specific heat is $\cdot 09515$ are mixed with an unknown weight of another substance, at a temperature of 20°C ., whose specific heat is $\cdot 615$. The resulting temperature is 18°C . What was the weight of the second substance used in the experiment?

26. An observer on immersing his thermometer into a mass of water, finds that the mercury in the tube sinks down into the bulb: how could he determine the temperature of the water without the use of another thermometer? Illustrate your answer by an example.

CHAPTER V.

EXPANSION OF SOLIDS.

53. Experiments on Expansion. We have already said that most bodies expand as their temperature rises, we proceed to examine in more detail the expansion of solid bodies. The following experiments illustrate this.

EXPERIMENT (7). Take a short rod of metal about 10 cm. in length with flat ends, and make a gauge out of thin sheet brass or some other material to fit the rod exactly. In fig. 21 *AB* is the rod, *CD* the gauge; heat the rod in a Bunsen flame. The gauge will no longer fit it but is too short.

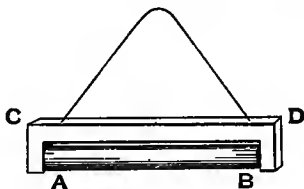


Fig. 21.

The same experiment may be performed with a sphere which has been turned so as just to pass when cold through a metal ring. On heating the sphere it will no longer pass through the ring.

EXPERIMENT (8). Take a rod of metal from 30 to 50 cm. in length. Let one end *B* rest on a horizontal glass plate while the other is fixed in some suitable manner as by the weight at *A* in fig. 22. Place a fine needle between the rod and the glass plate in such a way that if the rod moves it will cause the needle to roll on the plate.

Pass the point of the needle through a straw or a strip of thin aluminium foil to serve as a pointer. A small motion of the needle causes the end of the pointer to move over a considerable distance. Arrange under the rod a Bunsen gas-burner. On lighting this the rod is heated and expands. The end *A*

being fixed, the motion of the end *B* causes the needle to roll

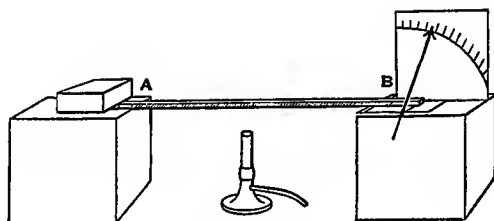


Fig. 22.

on the plate and moves the pointer. On removing the gas-burner the rod cools and the pointer comes back to its original position.

EXPERIMENT (9). *To shew that different bodies expand differently when raised to the same temperature.*

Let a gauge such as that described in Experiment 7 above be made of brass and an iron rod filed so as just to fit it when cold, then if the two be heated by passing them a few times through the flame of a Bunsen burner it will be found that the iron rod will drop out; the brass expands more than the iron.

Or again, take two bars, one of iron, the other of brass, of the same size, about 20 cm. long, 2 cm. broad and .1 cm. thick, and rivet them together with a number of rivets. Place them over the Bunsen gas-burner: they become curved, the iron being on the inside of the curve, the brass outside: brass expands more than iron.

54. Linear Expansion. The amount by which a rod of metal increases in length for a moderate rise of temperature, say 100°C ., differs as we have just said for different metals, but is in all cases very small. A rod of iron 1 metre long would increase in length, if raised through 100°C . in temperature, by about .12 cm.; the increase in length of a rod of brass of the same length would be about .18 cm.

Now experiment shews that the increase in length is

proportional to the original length and to the change in temperature. Thus for the same change of temperature 100°C . an iron rod 200 cm. long would increase by $2 \times \cdot 12$ or $\cdot 24$ cm., while a rod 50 cm. long would increase by $\frac{1}{2} \times \cdot 12$ or $\cdot 06$ cm.; again if the rod were raised through 200°C . the increase would be $\cdot 24$ cm., if through 1000°C . it would be $1\cdot 2$ cm. We are thus led to the following definition.

55. Definition of Coefficient of Linear Expansion. *The ratio of the increase in length produced by a rise of temperature of 1° to the original length is called the coefficient of linear expansion of a rod.*

Suppose a rod l_0 cm. in length is heated through t° and let its length become l cm., let α be the coefficient of linear expansion. Then the increase of length for t° is $l - l_0$, thus the increase for 1° is $(l - l_0)/t$ and the ratio of this to the original length is $(l - l_0)/l_0 t$,

$$\therefore \alpha = \frac{l - l_0}{l_0 t}.$$

Hence

$$l - l_0 = l_0 \alpha t, \\ \therefore l = l_0 + l_0 \alpha t = l_0 (1 + \alpha t).$$

Thus if we know the original length l_0 , the rise¹ of temperature t and the coefficient of linear expansion α , we can find the new length l .

We may put the definition in another form thus; let us suppose that $l_0 = 1$ cm. and that $t = 1^{\circ}$, then

$$\alpha = \frac{l - l_0}{l_0 t} = l - 1.$$

Now $l - 1$ is the increase in length of unit length, produced by a rise of 1° .

Hence we may say that the coefficient of linear expansion is the increase produced by a rise of temperature of 1° in a length of 1 centimetre.

¹ It must be noted that t is the rise of temperature: if T_0 is the initial temperature, T the final, then $t = T - T_0$, so that we may write

$$l = l_0 \{1 + \alpha (T - T_0)\}.$$

We may obtain the above formula from this definition thus :

the increase of 1 cm. for 1° is α ,
 \therefore the increase of l_0 cm. for 1° is αl_0 ;
 and the increase of l_0 cm. for t° is $\alpha l_0 t$.

Thus since the new length is the original length together with the increase for t° we have

$$l = l_0 + l_0 \alpha t = l_0 (1 + \alpha t).$$

Example. The coefficient of expansion of brass is $\cdot 0000189$. A metre scale is correct at 0°C . What error will be committed in measuring a length of 1 metre with it at 20°C .?

A length of 1 cm. expands by $\cdot 0000189$ cm. for 1° .

Thus the increase of 100 cm. for 1° is

$$100 \times \cdot 0000189 \text{ or } \cdot 00189 \text{ cm.},$$

and for 20° it is $\cdot 0378$ cm.

Hence the error is $\cdot 0378$ cm.

56. Measurement of Expansion. In order then to measure a coefficient of linear expansion we must measure both the small increase of length produced in a rod of measured length by a rise of temperature and the rise of temperature. Now the increase in length is very small; if it be measured directly very delicate instruments are required; we can however magnify the apparent increase in a known proportion as in the following experiment.

57. EXPERIMENT (10). *To measure the coefficient of linear expansion of a metal.*

AB , fig. 23, is a stout wire of the metal about 1 metre long hanging vertically from a firm support A . A long horizontal lever CBD is attached to the other end of the wire, C is the fulcrum and is secured to the same support as the upper end of the wire; this is shewn to a larger scale in fig. 23 (a). The distance CB is about $\cdot 5$ cm. and the length CD about 50 cm. These distances must be measured with care.

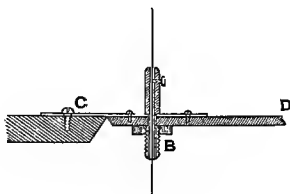


Fig. 23 (a).

If the wire expands B moves down and the end D of the lever moves, but the motion of D with the numbers given is one hundred times that of B . By means of a scale the motion of D is measured.

A vertical jacket tube EF of glass or brass closed with corks at the top and bottom surrounds the wire which passes freely through short lengths of glass tubing inserted in the corks at E and F . Two other tubes are inserted into the jacket and by their aid steam can be passed through the jacket raising the temperature of the wire to 100° .

To make an experiment the temperature of the wire t is read by a thermometer, and its length l_0 measured with an ordinary metre scale. The position of D on the scale is read. Steam

is then allowed to pass through the jacket; the wire expands and in consequence D comes to D' : measure the distance DD' , let it be d ; the temperature is now 100° , so that the rise in temperature is $(100 - t)$. The increase in length is BB' . Now from the similar triangles CBB' , CDD' , fig. 24, we have

$$\frac{BB'}{CB} = \frac{DD'}{CD},$$

$$\therefore BB' = DD' \times \frac{CB}{CD} = d \times \frac{CB}{CD}.$$

The lengths CB and CD are determined by measurement of the lever, in the instrument described we have

$$CB/CD = 1/100.$$

$$\therefore BB' = \frac{d}{100}.$$

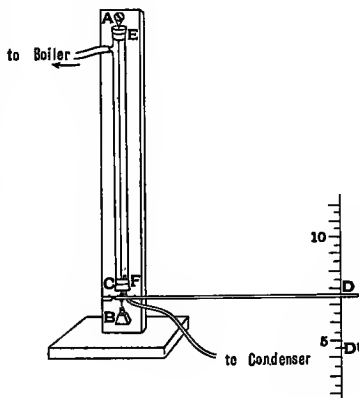


Fig. 23.

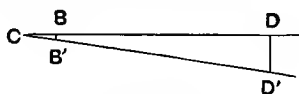


Fig. 24.

Now to find α the coefficient of expansion we have to divide the increase in length by the original length and by the rise in temperature: we thus get

$$\alpha = \frac{d}{100 \times l_0 (100 - t)}.$$

If the wire be copper 120 cm. long and t be 15°C. , we should find $d = 18$ cm. approximately.

Thus

$$\alpha = \frac{18}{100 \times 120 \times 85} \\ = \cdot 0000176.$$

58. Lavoisier's Method. In the apparatus used by Lavoisier and Laplace the rod of metal is placed in a horizontal position and rests on rollers in a trough containing oil or water which can be heated. The lever BD , fig. 25, is vertical; one end rests against the rod at B . A telescope DE is fixed to the lever at D with its axis horizontal; and a mark S on distant scale is viewed through the telescope. As the rod expands DB is turned into the position DB' , and a different mark S' on the scale is brought into view.

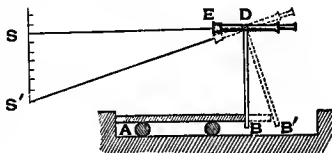


Fig. 25.

If the distances BD and DS be known, we can find BB' the extension, for we have

$$BB' = SS' \cdot \frac{BD}{DS},$$

and from this the coefficient of expansion can be found¹.

¹ In fig. 25 the expansion BB' is for the sake of clearness enormously exaggerated.

The measurement can also be made by fixing a mirror M on an axis which can be made to turn by a piece of fine silk attached to the lever at D , fig. 26, and passing over a smooth

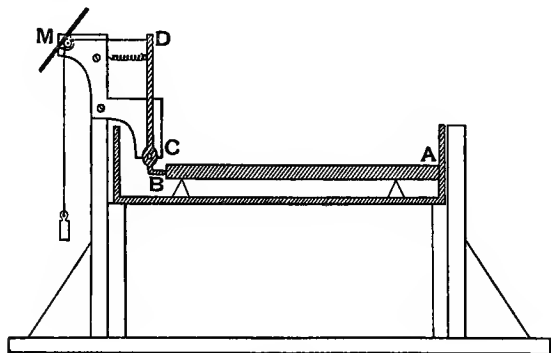


Fig. 26.

pulley. Light from a distant lamp falls on the mirror and is reflected on to a vertical scale. As the bar expands the mirror rotates and the reflected beam moves over the scale, the angle through which the beam turns being twice that described by the mirror¹.

Thus in this case we have, referring to fig. 25,

$$\frac{SS'}{SD} = \frac{2BB'}{DB},$$

$$\therefore BB' = \frac{1}{2} SS' \times \frac{DB}{SD}.$$

In order to keep the end B of the lever pressed up against the bar, a stiff spring is attached to the end D and to the frame of the apparatus.

Example. In an apparatus in which the short arm CB of the lever is 1 cm., and the distance between the mirror and scale 200 cm., when a rod of copper 50 cm. long is put in and heated from 15° to 100° the spot moves through 29 cm.: find the coefficient of linear expansion of copper.

¹ See *Light*, § 24.

In this case the ratio of the short arm of the lever to the distance between the mirror and scale, or BC/DS , is $\frac{1}{2 \cdot 100}$.

$$\begin{aligned}\therefore \text{Increase in length} &= \frac{1}{2 \cdot 100} \times 29 \text{ cm.} \\ &= \cdot 0725 \text{ cm.}\end{aligned}$$

$$\therefore a = \frac{\cdot 0725}{50 \times 85} = \cdot 000017.$$

59. Micrometer Method of measuring Expansion. Various methods have been devised for the direct measurement of the small increase in length produced by expansion. The following is one which may be employed.

EXPERIMENT (11). *To measure the coefficient of linear expansion of a rod with a micrometer.*

A micrometer scale is fitted in the eye-piece of a microscope. This consists of a very fine scale engraved on glass; each division of the scale being perhaps $\frac{1}{4}$ of a millimetre. The microscope is then focussed on a similar scale and the magnified image of this scale is seen in coincidence with the micrometer. The number of divisions of the micrometer which coincide with one division of the magnified scale are then counted. This determines the magnifying power of the object glass of the microscope; let us suppose this is found to be 25. Thus a distance which we know to be $\frac{1}{4}$ mm. appears to cover 25 divisions when viewed through the microscope, so that 1 division of the eye-piece scale corresponds to $\frac{1}{100}$ mm. in the object viewed.

Suppose then we are looking through the microscope at some fine mark which we see coincident with a division in the eye-piece scale and that this mark is moved so that its image appears to cross a number of divisions n , which we can count, of the eye-piece scale. We know that the mark has moved through $n/100$ mm. We are thus able to measure easily a very small displacement of the mark and can apply this to measure the expansion of a rod. For this purpose take a rod about 1 metre long with a fine mark engraved on it near each end. Measure with a scale the distance between the marks, let it be l_0 cm. Place it in a steam jacket through which steam can be passed, allowing the ends to project through corks in such a way that the marks are just visible

outside the tube, and arrange a microscope with a micrometer scale in the eye-piece to view each mark. Place a thermometer in the tube alongside the rod and take the temperature, let it be t° ; note the position of the mark on each of the micrometer scales. Pass steam from a boiler through the jacket. The marks will appear to move across the scales and after a time will take up a stationary position again when the rod has come to the temperature of the steam; let a and b be the number of divisions of the scales through which the marks have moved. Then, if the divisions of the scales be $\frac{1}{4}$ mm. and the magnifying power of the object glass 25, as described above, the increase in the length of the rod will be

$$\frac{a+b}{100} \text{ millimetres or } \frac{a+b}{1000} \text{ cm.}$$

The original length is l_0 cm., thus the increase in length of unit length is $(a+b)/1000l_0$, and this is for a rise of temperature of $100-t$. Hence the coefficient of expansion is

$$\alpha = \frac{(a+b)}{1000l_0(100-t)}.$$

60. Sources of error in the experiments.

Besides those we have noted there are other sources of error common to the experiments described in Sections 56 to 59.

(1) We suppose that the distance between the microscopes in Section 59 and the distance between the point of suspension and the fulcrum in Section 56 are not altered by the change of temperature. Neither of these statements is strictly accurate, but if the microscopes are fastened down to a stout wooden or stone table, the error will be very small, and the same will apply to Experiment 10 if the support and fulcrum are properly secured.

(2) A portion of the wire in Experiment 10, and of the rod in Experiment 11, is outside the steam jacket, so that the temperature of the whole is not accurately 100° . This can be made small by making the exposed parts as short as possible.

The method of Lavoisier and Laplace described in Section 58 is free from both objections.

The experiment described in Section 59 is a modification of the method employed by Roy and Ramsden. In their apparatus the whole of the bar whose expansion was to be measured was in an oil bath, while the bar to which their reading microscopes were secured was kept in ice. The errors we have just been describing were thus avoided.

61. Coefficients of Expansion. The approximate values of the coefficient of linear expansion of various substances are given in the following table.

Aluminium	·0000222	Platinum	·0000089
Brass	·0000189	Silver	·0000194
Copper	·0000167	Steel	·0000110
Gold	·0000142	Zinc	·0000298
Iron	·0000117	Ebonite	·0000770
Lead	·0000280	Glass	·0000089

These values are only approximate, for the coefficients depend on the state of the material, and, in the case of a composite substance such as glass or ebonite, on the exact composition of the specimen.

It will be noticed that the coefficient for platinum is the same as for glass. This is very important, for in consequence platinum wire can be hermetically sealed into glass. It would be impossible to do this with copper wire, for the glass and the copper must be heated red hot; in cooling down the glass becomes fairly rigid at about 400°C. , but the copper contracts for a given change of temperature nearly twice as much as the glass and so becomes smaller than the hole in the glass through which it passes and cracks the glass.

It will also be observed that the coefficients of expansion are all very small and in consequence their exact determination is a matter of some difficulty.

62. Superficial and Cubical Expansion. Up to the present we have been dealing with expansion in one direction only, but if a body be heated it expands in all directions, it increases—with some few exceptions—in surface and in volume.

We have therefore to consider the surface or superficial expansion and the volume or cubical expansion of a body.

Now it is found that the increase in surface or in volume is proportional to the original surface or volume respectively and to the change in temperature.

We may thus define the coefficients of superficial and of cubical expansion.

63. Definition. *The Coefficient of $\left\{ \begin{smallmatrix} \text{superficial} \\ \text{cubical} \end{smallmatrix} \right\}$ expansion is the ratio of the increase in $\left\{ \begin{smallmatrix} \text{surface} \\ \text{volume} \end{smallmatrix} \right\}$ produced by a rise of temperature of 1° to the original $\left\{ \begin{smallmatrix} \text{surface} \\ \text{volume} \end{smallmatrix} \right\}$.*

From the definition we may obtain formulae similar to that found in § 55 for the coefficient of linear expansion. For if S_0 be the original area, S the area when the temperature has been raised t° , and β the coefficient of expansion, the increase in area for 1° is $(S - S_0)/t$, and β the ratio of this to the original area is given by the equation

$$\beta = \frac{S - S_0}{S_0 t},$$

$$\therefore S = S_0 (1 + \beta t).$$

Similarly if γ be the coefficient of cubical expansion, V_0 and V the volume and t the rise of temperature,

$$\gamma = \frac{V - V_0}{V_0 t},$$

$$V = V_0 (1 + \gamma t).$$

We may also as in Section 55 state that

The Coefficient of $\left\{ \begin{smallmatrix} \text{superficial} \\ \text{cubical} \end{smallmatrix} \right\}$ expansion is the increase in $\left\{ \begin{smallmatrix} \text{area} \\ \text{volume} \end{smallmatrix} \right\}$ of unit $\left\{ \begin{smallmatrix} \text{area} \\ \text{volume} \end{smallmatrix} \right\}$ produced by a rise of temperature of 1° .

64. Relation between Coefficients of Expansion.

We may shew in the following way that there is a relation between these coefficients of expansion. Consider a square each side of which is 1 cm. Its area is 1 square cm. Let its temperature be raised t° . The area increases to $1 + \beta t$, but each side increases to $1 + \alpha t$. Thus the new area is $(1 + \alpha t)^2$,

$$\therefore 1 + \beta t = (1 + \alpha t)^2 = 1 + 2\alpha t + \alpha^2 t^2.$$

Now since α is very small for most materials, $\alpha^2 t^2$ is very

small compared with at unless t is very large; for example if $t = 100^\circ$ and the square be made of copper for which

$$a = .000016,$$

we have

$$at = .0016,$$

$$a^2t^2 = .00000256,$$

and a^2t^2 is so small that we may omit it: we have then

$$1 + \beta t = 1 + 2at.$$

$$\text{Thus } \beta = 2a.$$

Again consider a cube each side of which is 1 cm. Its volume is 1 c.c. On heating it t° its volume becomes $1 + \gamma t$ cubic centimetres; but each edge becomes $1 + at$ cm. in length.

Thus the new volume is $(1 + at)^3$ c.cm.

$$\text{Hence } 1 + \gamma t = (1 + at)^3 = 1 + 3at + 3a^2t^2 + a^3t^3.$$

Now we have seen that we may neglect a^2t^2 and *a fortiori* a^3t^3 compared with at . Thus we have

$$1 + \gamma t = 1 + 3at,$$

$$\therefore \gamma = 3a = \frac{3}{2}\beta.$$

If then the coefficient of linear expansion be known, the coefficients of superficial and of cubical expansion can be found from the above.

The exact meaning of the neglect of the terms involving a^2 and a^3 may be made more clear by the following.

Let $ABCD$ be a square each side of which is 1 cm. and let $A'B'C'D'$ be the same square when its temperature has been raised 1° . Produce AB and CB to meet $C'B'$ and $A'B'$ in F and E respectively.

Then the side DA , 1 cm. in length, has increased by an amount AA' , thus AA' is equal to a . DC also 1 cm. has increased by CC' , hence CC' is equal to a .

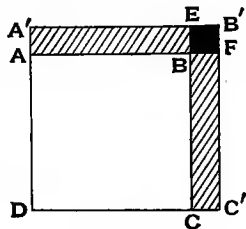


Fig. 27.

Hence also $BE = BF = a$.

Hence area of parallelogram $AE = a$.

area of parallelogram $CF = a$.

area of square $EF = a^2$.

And the increase of area of the square centimetre $ABCD$ is rectangle AE + square EF + rectangle CF or $2a + a^2$.

In neglecting a^2 compared with a we are neglecting the square EF compared with the rectangle AE ; if we remember that AB is 1 cm. while BE or EF for copper is less than two hundred thousandths of a centimetre, it is clear that we may do this and say that the increase in area is the sum of the rectangles AE and CF or $2a$. But β is the increase in area of 1 square cm.

$$\therefore \beta = 2a.$$

65. Linear and Cubical Expansion. Again consider a cube (shewn in fig. 28 with dark lines) each edge of which is 1 cm.; let its temperature be raised 1° , the figure will become a cube having each edge equal to $(1 + a)$ cm. This cube is made up of the original cube together with three square slabs each of which is 1 square cm. in area and a cm. thick, three rectangular strips each 1 cm. high on a base of a^2 square cm. and a small cube whose edge is a cm.

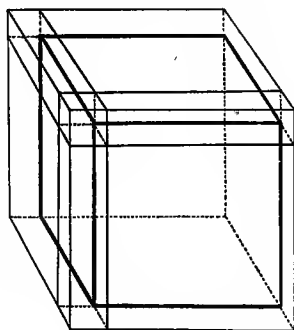


Fig. 28.

Thus the new volume is

$$1 + 3a + 3a^2 + a^3.$$

In neglecting the terms in a^2 and a^3 , we are neglecting the volumes of the small cube and of the rectangular strips compared with that of the slabs.

Now by definition the new volume is

$$(1 + \gamma) \text{ c. cm.}$$

Hence we have $1 + \gamma = 1 + 3\alpha$
 $\gamma = 3\alpha$.

The coefficient of cubical expansion is very often called the coefficient of dilatation.

The coefficient of superficial expansion is usually found by determining the coefficient of linear expansion and multiplying by 2.

Methods have been devised for finding the coefficient of cubical expansion directly; one of these is given in § 80. The values so found agree with the number obtained by multiplying the coefficient of linear expansion by 3.

66. Examples on Expansion.

A difficulty is sometimes met with in calculating the change in volume of the interior of a hollow vessel, the bulb of a thermometer for example, which expands by heat. Let us suppose that the bulb is filled with the same material as the walls are composed of. When it is heated the whole expands together; the interior of the bulb will still clearly remain full; the new volume will be found from the old volume by the use of the coefficient of cubical expansion of the material of the walls.

Example. A glass bottle holds when full at a temperature of 0°C . 500 c.cm. of water; how many c.cm. will it hold when heated to 100°C .?

The coefficient of linear expansion of glass is $\cdot 0000089$, therefore the coefficient of cubical expansion is $3 \times \cdot 0000089$ or $\cdot 0000267$.

Thus the increase in volume of 500 c.c. for a rise of 1° is

$$500 \times \cdot 0000267 \text{ or } \cdot 01335 \text{ c.cm.}$$

Hence the increase for 100° is $100 \times \cdot 01335$ c.c. or $1\cdot 335$ c.c.

Thus the new volume is $501\cdot 335$ c.cm.

67. Relation between Density and Coefficients of Expansion. We have already seen that the density of a homogeneous body is measured by the ratio of its mass to its volume. Now when a body is heated its mass is not altered but in general its volume is increased, its density therefore is diminished in exactly the same proportion as its volume is increased. Thus if V_0 , V be the volumes, ρ_0 , ρ the corresponding densities, since the mass is constant we have $V\rho = V_0\rho_0$,

$$\therefore \frac{\rho_0}{\rho} = \frac{V}{V_0}.$$

Again if γ be the coefficient of cubical expansion and t the rise of temperature

$$\gamma t = \frac{V}{V_0} - 1 = \frac{\rho_0}{\rho} - 1,$$

$$\therefore \gamma = \frac{\rho_0 - \rho}{\rho t}.$$

Now $\frac{\rho_0 - \rho}{t}$ will be the change of density produced by a rise of temperature of 1° . We may therefore say that the coefficient of cubical expansion is the ratio of the decrease of density produced by a rise of 1° to the final density.

Again from the above we have

$$\rho_0 = \rho (1 + \gamma t).$$

Thus

$$\rho = \rho_0 \frac{1}{1 + \gamma t}.$$

Now on dividing 1 by $1 + \gamma t$ we get

$$1 - \gamma t + \gamma^2 t^2 - \gamma^3 t^3 + \dots$$

and we have already seen that for most solid substances γ is so small that we may neglect γ^2 , γ^3 etc.

Hence we have $\rho = \rho_0 (1 - \gamma t)$,

and this form is more readily employed in numerical calculation.

Example. The density of silver at 0° is 10.38 gramme per c.cm., find its density at 800° .

The value of γ for silver is $3 \times .0000194$ or $.0000582$.

Hence the value of γt is $.0456$.

$$\therefore \rho = 10.32 (1 - .0456)$$

$$= 10.32 - .47$$

$$= 9.85 \text{ gramme per c.cm.}$$

In what precedes, the coefficients of expansion have been defined as the change per 1° C. in some quantity such as a length.

Since 1° Fah. is $\frac{5}{9}$ of 1° C. the change per 1° Fah. will be $\frac{5}{9}$ of the change per 1° C., that is, coefficients of expansion per 1° Fah. will be found by multiplying the values given in the Table by $\frac{5}{9}$.

68. Practical consequences of Expansion. These are very varied, some of them have already been alluded to, others may be mentioned here. The standards of length are

bars of brass or bronze on which certain fine marks have been engraved. The standard yard for example is the distance between two such marks on platinum plugs let into a bronze bar, but this distance depends on the temperature and hence the temperature at which the distance is one yard needs to be defined. The English standard temperature is 62° Fah. or $16\frac{2}{3}^{\circ}$ C. The standard metre again is correct at 0° C. While making accurate measurements of length, great precautions have to be taken to secure uniformity of temperature.

The rate of a clock depends on the length of the pendulum between the point of suspension and a point called the centre of oscillation; if the pendulum consist merely of a metal rod with a bob at the end, as the temperature rises the rod gets longer and the clock loses and *vice versa*.

69. Graham's Mercurial Pendulum. This instrument has usually an iron rod, but the bob consists of a glass vessel containing mercury. The coefficient of dilatation of mercury is much greater than that of iron. As the temperature rises the iron expands and this lowers the centre of oscillation, but the mercury expands more, thus raising the centre, and the pendulum is constructed so that the drop from the one cause just balances the rise due to the other.

70. Harrison's Gridiron Pendulum. Let AB , CD be two parallel rods of different material fastened together at the ends B and C and suppose A is fixed. If the temperature rises the end B is lowered, but owing to the expansion of CD , D is raised, and if CD be of more expansible material than AB the rise of D may be just equal to the fall of B , so that the distance AD may remain unchanged; let the lengths of AB and CD be l and l' and let α , α' be the coefficients of expansion.

Then if the temperature is raised 1° B moves down a distance $l\alpha$, but D moves up, relative to C , a distance $l'\alpha'$. Thus if $l\alpha = l'\alpha'$ the distance AD will not change. The necessary condition therefore is that the lengths of the rods should be inversely proportional to their coefficients of expansion.



Fig. 29.

If now we take an arrangement such as that shewn in fig. 30, the rods¹ a, a, b being of one material and the rods c, c of another; the expansion of the rods a, a, b will lower the bob G , while that of c, c will raise it. The distance it will be lowered for a rise of 1° will be $(a+b)a$ while the distance it is raised will be ca' ; if it be possible to make these equal the distance OG will not change and the pendulum will be compensating. With five rods as shewn and the materials ordinarily used, iron and brass, this is not possible, for we should require to have

$$\frac{a+b}{c} = \frac{a'}{a} = \frac{189}{117} = \frac{3}{2} \text{ approximately;}$$

and since $a+b$ is necessarily greater than twice c this is impossible, but by increasing the number of rods the result can be secured.

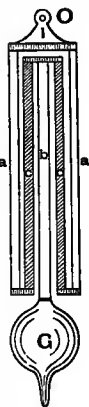


Fig. 30.

With the two materials iron and brass, it will need four brass rods and five iron rods.

71. Balance Wheel of a Watch. The rate of a watch depends mainly on the balance wheel, as the temperature rises this expands unless it is compensated and the watch loses time, for a large wheel will oscillate under a given force, which is supplied by the elasticity of the hair spring, more slowly than a smaller wheel.

Compensation is secured by making the rim in three sections, each section is carried by a single arm and is made of two materials; as the temperature rises the arms expand and their extremities move outwards, but the arcs become more curved, for the more expansible metal is on the outside, so that their free ends move inwards. By properly adjusting the mass of the rim, and this is done by attaching a number of small screws to it, its time of oscillation can be made constant in

¹ The rods c and a are duplicated to secure symmetry, the theory so far as the expansion is concerned would be the same if they were single.

spite of variations of temperature. The elasticity of the hair spring diminishes slightly as the temperature rises, in consequence the force acting on the wheel gets less and so the wheel has to be overcompensated to make up for this.

72. Metallic Thermometers. The unequal expansibility of different metals is made use of to measure temperature in Breguet's metallic thermometer. The instrument consists of very thin strips of silver, gold and platinum, which are rolled together so as to form a narrow ribbon. This is then wound into a spiral, the silver being inside, and the platinum outside. The upper end of the spiral is fixed, and the lower end is attached to a pointer which moves over a horizontal scale as the spiral unwinds. If the temperature rises, the silver expands more than the platinum, the spiral unwinds, and a very small change of temperature is sufficient to produce considerable motion of the pointer.

The instrument may be made more sensitive by substituting a mirror for the pointer and reflecting a beam of light from it on to a distant scale.

73. Effects of Expansion. In all engineering work in which metal is employed, special precautions have to be taken to meet difficulties arising from expansion. Thus long iron girders such as are used in bridges or roofs are not rigidly secured to the masonry on which they rest, but are free to move slightly with changes of temperature. An iron girder 50 ft. in length will increase about $\cdot 12$ of an inch for a rise of temperature of 20°C .

The rails on a railway line are placed at some little distance apart, the holes in the fish-plates being slotted, to allow for expansion; iron gas and water pipes have telescopic joints; furnace bars are made with bevelled ends, and fit loosely into the brickwork of the furnace for the same reason.

Very considerable force may be exerted by a hot body in cooling. This may be shewn by the following experiment.

EXPERIMENT (12). *To shew the stress due to change in temperature.*

Take a bar of iron about 30 cm. long of square section, each side of the square being from 2 to 3 cm.

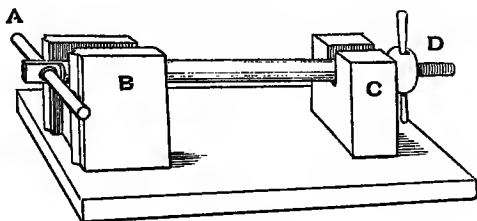


Fig. 31.

One end *D* of the bar is turned down and a strong screw cut on it. A nut works on this screw. The other end of the bar is forged to the shape shewn at *A*, and through this a hole about 1 cm. in diameter is drilled; the edges of this hole are *V* shaped, the bar can fit tightly between two stout vertical supports *B*, *C*. Two *V* shaped projections some 3 or 4 cm. apart are attached to *B* and a short piece of cast-iron rod passes loosely through the hole in the bar and is pressed up against these *V*'s by the nut working against the support *C*. The arrangement is shewn fully in fig. 31 (*a*). Heat the bar red hot. Pass one of the short iron rods through the hole, and place the bar on the supports screwing the nut up tight. As the bar cools it contracts, and the force due to the contraction is sufficient to break the short piece of cast iron which is pressed up against the *V*'s.

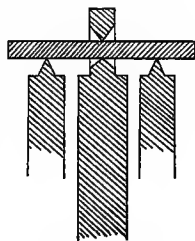


Fig. 31 (*a*).

We may calculate the force exerted by the bar in cooling through 100°C . thus. Each centimetre would contract on cooling by $\cdot 0011$ cm.; if then it is prevented from contracting by the application of force, the force must be sufficient to produce an extension of $\cdot 0011$ cm. in a length of 1 cm. This force has been measured, if the area of the cross section of the bar be 1 sq. cm. the force necessary is found to be about 2,000,000 grammes' weight.

If each side of the bar be 2.5 cm. in length, we must multiply this by 6.25, the area of the bar in sq. cm., and find thus as the force exerted, 12,500 kilograms' weight.

The shrinking of a body in cooling is made use of in various ways. Thus the tyre is put on to a wheel red hot, in cooling it contracts and holds the wooden rim together. Large guns are made by coiling a bar of iron into a close spiral, the turns of the spiral are welded together and a cylinder is thus formed. A number of such cylinders are placed over an inner tube when red hot, each cylinder, as it contracts, compresses those below it, and the stress produced in the breech by the combustion of the powder is thus distributed throughout the material of the gun more evenly than would be possible if the gun were turned out of a single piece of the metal.

EXAMPLES.

EXPANSION OF SOLIDS.

1. The volume of a mass of copper at 50°F . is 1 c. ft.; find its volume at 0°C . and at 100°C .

2. A glass vessel holds 2 litres at 15°C . How much will it hold at 25°C .?

3. The specific gravity of iron at 0°C . is 7.76; find its value at 100°C . referred to water at 0°C .

4. A copper rod 125 cm. long at 0°C . expands by .209 cm. when raised to 100°C . Find the coefficient of expansion of copper.

5. An iron bridge is 250 feet long. Find its variation of length between the temperatures of -20°C . and 50°C .

6. A copper rod is 150 cm. long at 0° ; at what temperature will it have increased by 2 mm.?

7. A rod of iron and a rod of zinc are each 2 metres long at 0°C . How much longer is the zinc than the iron at 50°C .?

8. A gridiron pendulum has 5 iron rods each 1 metre in length and 4 brass rods. Find the length of each brass rod.

9. An iron clock pendulum makes 86,405 oscillations one day; at the end of the next day the clock has lost 10 seconds; find the change in temperature.

10. The iron rails on a railway line are each 6 feet long. What space must be left between two consecutive rails to allow space for expansion if the temperature may range over $50^{\circ}\text{C}.$?

11. The area of an iron plate at $0^{\circ}\text{C}.$ is 50 sq. cm., find its area at $50^{\circ}\text{C}.$

12. What is meant by the statement: The coefficient of linear-expansion of copper is $\cdot000017$? If a copper rod is 20 yards long at $0^{\circ}\text{C}.$, how much longer will it be at $90^{\circ}\text{C}.$?

13. If the mean coefficient of expansion of water between 0° and 50° be $\cdot000236$, find the weight of water displaced by a brass cube whose side at 0° is 1 centimetre in length when the water and the cube are both at $50^{\circ}\text{C}.$

14. A cylinder of iron, 20 inches long, floats vertically in mercury, both being at the temperature $0^{\circ}\text{C}.$ If the common temperature rises to $100^{\circ}\text{C}.$, prove that the cylinder will sink $\cdot163$ inches.

[Specific gravity of iron at $0^{\circ}\text{C}.$ = 7.6,

„ „ mercury „ = 13.6,

cubical expansion of mercury between 0° and $100^{\circ}\text{C}.$ = $\cdot018153$,

linear „ iron „ „ = $\cdot001182$.]

15. The correct length of a steel chain at $0^{\circ}\text{C}.$ is 66 feet; express as a decimal of an inch the change in its length produced by its temperature being raised to $20^{\circ}\text{C}.$ The coefficient of linear expansion of steel is $\cdot0000116$.

16. A brass pendulum which keeps correct time at $0^{\circ}\text{C}.$ loses 16 seconds a day at $20^{\circ}\text{C}.$; find the coefficient of expansion of brass.

CHAPTER VI.

DILATATION OF LIQUIDS.

74. Definition of Coefficient of Dilatation. Most liquids, like most solids, expand when the temperature is raised and the coefficient of cubical expansion or of dilatation of a liquid is defined thus:

Definition. *The coefficient of dilatation is the ratio of the increase in volume, for a rise of temperature of 1° , to the volume at 0° C.*

It may also be defined, as in § 67 with reference to the density, as the ratio of the decrease in density due to a rise in temperature of 1° to the final density.

In dealing with solids the coefficient of dilatation has been defined as the ratio of the increase in volume per degree rise of temperature to the volume at the lower temperature; it would have been more accurate to have used throughout the volume at 0° C. In the case of solids however the increase in volume is so small that this difference is inappreciable in the result. Thus 10 c.cm. of iron at 0° become 10.0037 c.cm. at 15° ; if we suppose this heated still further the increase of volume per 1° will be .000355 c.cm. According to the definition the coefficient of expansion is .000355/10.004; more strictly it is .000355/10, for the volume at 0° is 10 c.cm. The two differ inappreciably.

In liquids however the dilatation is greater, and the error made in dividing by the volume at the lower temperature instead of the volume at 0° may be appreciable.

Thus 10 c.cm. of alcohol at 0° is 10.16 c.cm. in volume at 15°C . The increase in volume per 1° of temperature is .011 c.cm.; if we divide by 10 we get as the coefficient of expansion the value .00110, while if we use 10.16—the volume at the lower temperature—as the divisor we find for the coefficient .00108; and the difference between these numbers is sufficient to be considered.

The coefficient of dilatation of a gas again is much greater than that of a liquid, and in dealing with the expansion of gases it becomes necessary to define the coefficient strictly with reference to the volume at 0° .

75. Dilatation or Cubical Expansion of a Liquid.

In considering a liquid we have only to deal with dilatation or cubical expansion, linear expansion does not concern us. Thus, if we have a solid rod such as $ABCD$ fig. 32, and heat it, the rod increases in length, breadth and thickness to $A'B'C'D$, being equally free to expand in all directions; we can distinguish between the increase in length CC' and the increase in diameter AA' . If we now take a liquid it must be contained in some vessel; let us suppose for the present that the vessel does not expand¹. The column cannot increase in thickness, and the whole increase in volume shews itself as an increase in the length of the column, the transverse expansion is prevented from taking place, and the liquid which would, if it were free to do so, fill the space corresponding to $AA'B'B$ in fig. 32 is pushed to the end of the

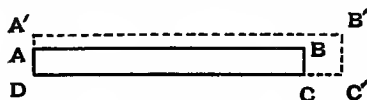


Fig. 32.

column increasing its length still further; the increase in length is proportional to the increase in volume, and from it the coefficient of dilatation may be found by dividing by the original length and by the change of temperature.

¹ We consider the effects due to the expansion of the vessel shortly.

76. Absolute and Relative Dilatation. In the above we have assumed that the containing vessel does not expand, and this is practically impossible.

In reality of course the tube or vessel in which the liquid is contained becomes larger when heated, there is some space corresponding to $AA'B'B$ in fig. 32, into which the liquid can expand, and the final length of the column depends in part on the expansion of the liquid and in part on that of the vessel.

To see how the two are connected consider the following argument; let us suppose it possible to heat the vessel without heating the liquid, and then to heat the liquid.

Take a bulb, fig. 33, with a long tube attached to it filled with liquid, and let the liquid stand at a height A in the tube. Suppose the temperature of the bulb raised without heating the liquid. The bulb increases in volume, the liquid remains unchanged; thus the level of the column sinks from A to B , and the volume of the tube between A and B measures the increase in volume of the bulb for the given rise of temperature; it is therefore proportional to the coefficient of expansion of the bulb.

Now let the liquid be heated to the same temperature as the bulb, it increases in volume, the column rises in the tube and finally comes to C . The volume between B and C is proportional to the coefficient of expansion of the liquid. Hence the volume between A and C is proportional to the difference between the coefficients of expansion of the liquid and vessel.

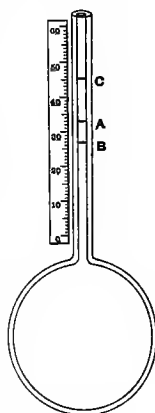


Fig. 33.

But the change in volume AC is what we should actually observe if we raised the temperature of the liquid and vessel together, it is the change in volume which is apparent to

us and is thus proportional to the *apparent coefficient of expansion* of the liquid.

We thus get the result that

Apparent coefficient of dilatation of a liquid = coefficient of dilatation of the liquid - coefficient of dilatation of the containing vessel.

The apparent expansion is the expansion which we observe directly without taking account of the changes in the containing vessel; it is often spoken of as the *relative expansion or dilatation*.

Definition. *The coefficient of dilatation of a liquid relative to its containing vessel is the ratio of the observed change in volume when both the liquid and the vessel are heated 1°C. to the original volume at 0°C.*

In determining the observed change in volume the expansion of the vessel is neglected.

Most methods of determining the coefficient of expansion of a liquid give us the relative or apparent expansion only.

To find the coefficient of dilatation we must add to the apparent coefficient the coefficient of expansion of the vessel.

It must be noticed that the change in density produced by a rise in temperature depends on the true coefficient of expansion, for the density is the mass of a unit volume of the liquid, and a change in density involves a change in the actual volume of the liquid not merely a change in its apparent volume relative to some vessel.

Thus by measuring the density of a liquid at different temperatures, we can find its coefficient of dilatation.

77. EXPERIMENT (13). *Absolute and apparent expansion of a liquid.*

Take a glass bulb some 5 or 6 cm. in diameter with a tube 2 or 3 mm. in diameter attached as in fig. 33. Fill it with

water¹. Attach a paper scale to the tube and fix it in a clip. When the temperature has become steady, note on the scale the height of the liquid in the tube. Take a beaker of warm water and suddenly immerse the bulb in the warm water, watching at the same time the level of the liquid in the tube. It will be found that the liquid at first sinks a few divisions on the scale and then rises, finally becoming steady some distance above its former position. The glass bulb acquires the temperature of this warm water some time before the liquid which it contains. The bulb therefore expands and the fall observed at first is due to this expansion. The liquid itself soon becomes heated, and the rise is due to its expansion. Since the liquid expands considerably more than the glass, the final rise is greater than the initial fall and the level of the liquid in the tube at the end is above its original position.

78. Different liquids expand by different amounts for the same rise of temperature. This is shewn by the following experiment.

EXPERIMENT (14). *To compare the apparent expansions of two liquids—such as water and alcohol—for a given rise of temperature.*

Take two thermometer tubes of equal bore terminating in bulbs of the same size. The diameters of the bulbs may conveniently be about 5 cm. and those of the tubes 3 or 4 mm. Fill one bulb and a portion of the tube with water, and the other with an equal volume of alcohol. Attach millimetre scales to both tubes. Place the two bulbs side by side in a vessel of cold water and note the levels at which the liquids stand. Transfer them to a vessel of warm water (the temperature of this water should not be above 55° C.), and note the new levels of the liquids. It will be found that the alcohol has risen through a greater distance than the water; determine the ratio of the distances through which the two

¹ To do this, warm the bulb gently, holding the open end of the tube under the water, then allow the bulb to cool. The atmospheric pressure forces some of the water into the bulb. Heat the bulb gradually, until this water boils, still keeping the end of the tube under the water; the steam escapes, carrying with it most of the air in the bulb, and the bulb is filled with boiling water and steam. Now allow the bulb to cool; the steam condenses and water is forced up the tube to fill the bulb.

columns have risen, since the volumes of the two fluids and the bores of the two tubes are respectively the same; this ratio will be the ratio of the mean coefficients of dilatation of the liquids between the given limits of temperature.

79. EXPERIMENT (15). *To determine the coefficient of apparent dilatation of alcohol in a glass vessel by means of an alcohol thermometer.*

Repeat carefully the Experiment of Section 77 with a bulb and tube containing alcohol, but in addition observe with a thermometer the temperatures of the two baths in which the bulb is placed; let them be t_1° and t_2° . Let l cm. be the distance the column rises for this rise of temperature, let the volume of the alcohol in the bulb at 0° C. be given as V c.cm. and let the area of the cross section of the tube be a sq. cm. Then the increase in volume for a rise of temperature of $t_2 - t_1$ is la c.cm.

Thus the increase of volume for 1° is $la/(t_2 - t_1)$, and dividing this by V the volume at 0° we find for the coefficient of apparent dilatation the value

$$\frac{la}{V(t_2 - t_1)}.$$

If the coefficient of expansion of the glass be known, by adding it to the apparent coefficient determined as above the true coefficient of dilatation of alcohol can be found.

The volume of the bulb and tube may be found thus. Warm the bulb gently and then allow it to cool with the open end of the tube under mercury. A thread of mercury will be drawn up into the tube; with care a thread some 6 or 8 cm. in length can be obtained without allowing any of the mercury to enter the bulb. Measure the length of this thread, let it be l cm. Warm the air in the bulb again and expel the mercury into a small weighed cup. Weigh the mercury, let its mass be m grammes. Since the density of mercury is 13.59 grs. per c.cm. the volume of the mercury is $m/13.59$ c.cm. and the area of the cross section of the tube, assuming it uniform, is $m/l \times 13.59$ sq. cm.

To find the volume of the bulb. Weigh the bulb and tube empty, then fill it with a liquid of known density, water will usually do, and weigh it again; let the mass of water contained be M grammes. Then taking the density of the water as 1 gr. per c.cm. the volume is M c.cm. Measure the length of tube occupied by water, multiply this length by the area of the tube and subtract the product from M . The result will be the number of cubic centimetres in the bulb.

80. EXPERIMENT (16). *To find the coefficient of dilatation of a liquid relative to glass by the weight thermometer.*

The weight thermometer consists of a glass tube (fig. 34) some 5 or 6 cm. long by about 1 cm. in diameter closed at one end. The other end is drawn out to a fine point and left open. Weigh the tube empty, fill it with the liquid whose coefficient of expansion is required. This is done as in § 77 by repeated heatings and coolings. When it is full leave it for a short time in a beaker of water, with its open end dipping into a small cup of the liquid, until it acquires the temperature of the water¹; let this be t_1° .

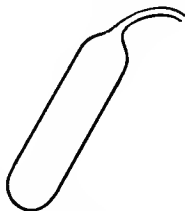


Fig. 34.

Dry the thermometer and weigh it again. The difference between these weighings gives the mass of liquid which fills the tube at t_1° ; let it be m_1 grammes.

Heat the thermometer to a higher temperature t_2° by putting it in hot water, or better, by suspending it in steam issuing from boiling water; some of the liquid is forced out; wipe this off with a piece of blotting paper, allow the whole to cool, the liquid contracts in the tube; when cold weigh it again, and by subtracting the mass of the empty tube find the mass of liquid which fills the bulb at t_2° ; let it be m_2 grammes.

Then, neglecting the expansion of the glass, we see that m_2 grammes at t_2° occupy the same volume as m_1 grammes at t_1° .

Hence at t_2° a given mass of liquid occupies m_1/m_2 of its volume at t_1° .

Subtracting from this the original volume at t_1° , we find that the increase of volume for a rise $(t_2 - t_1)$ is $(m_1/m_2 - 1)$ of the original volume.

Hence the coefficient of dilatation, being the ratio of the

¹ For very accurate work the beaker should contain ice not water, so that t_1 may be zero.

increase in volume per degree rise of temperature to the original volume, is

$$\frac{m_1 - m_2}{m_2 (t_2 - t_1)}.$$

This is of course the coefficient of dilatation relative to glass. If the coefficient of dilatation of the glass be known, we can get the true coefficient for the liquid. It is not easy however to determine the coefficient of the glass accurately, and the method of § 83 is used to find the true coefficient.

Example. In an experiment with a weight thermometer, 11.222 grammes of glycerine filled the thermometer at 15°; when the thermometer was raised to 100° C. it contained 10.765 grammes; find the coefficient of expansion of glycerine.

11.222 grammes at 15° occupy the same volume as 10.765 grammes at 100° C. Thus the volume of 1 gramme at 100° C. is $11222/10765$ (or 1.0425) of its volume at 15°. Thus the increase for 85° is .0425 of the original volume. Hence the coefficient of dilatation is .0425/85 or .00050.

81. Experiments with the weight Thermometer.

The weight thermometer may also be used to find the coefficient of dilatation of a solid in the following way. Take a piece of solid of such a form that it can be inserted into a weight thermometer before its end is drawn out. Determine the mass and volume of the solid. The volume is given by dividing the mass by the density, let it be 10 c.cm. Place the solid in the thermometer and draw out the neck to a fine point. Weigh the tube and solid. Fill the tube with liquid of known density and coefficient of dilatation; let it be glycerine, the density of which is 1.3 grammes per c.cm. and the coefficient of dilatation .00050. Determine the mass of liquid in the tube; suppose it to be 11.22 grammes and the temperature 0° C.

Raise the temperature to 100°; glycerine is expelled, partly because of the expansion of the liquid, partly because of that of the solid. Weigh the tube and liquid again and find the mass expelled, let it be .673 gramme; since the coefficient of dilatation of the glycerine is .0005 the mass expelled in consequence of the expansion of the glycerine will be approximately

$$.0005 \times 100 \times 11.22 \text{ or } .561 \text{ gramme.}$$

The difference of .112 gramme is due to the expansion of 10 c.cm. of the metal.

Now the volume of .112 gramme glycerine is very nearly $.112/1.30$ or .086 c.cm.

Thus the increase in volume of 10 c.cm. of metal for a rise of 100° C. is .086 c.c. The coefficient of dilatation therefore is

$$.086/10 \times 100 \text{ or } .00086.$$

82. Absolute Dilatation of a Liquid. To determine the true coefficient of dilatation of a liquid, we require to compare the densities of the liquid at two different temperatures. This can be done by the following method.

Take a tube of glass some 2 metres long and .5 cm. in diameter and bend it into the shape shewn in fig. 35. Place it with the two limbs AB , CD vertical, the arm BC being horizontal. Suppose that one limb AB and a portion of the arm BC is filled with one liquid and the other limb DC with a portion of the horizontal arm with a second. Let E be the junction of the two and let A and D be the tops of the columns of liquid in the two arms, let h , h' be the heights AB , DC of the two columns, ρ , ρ' the densities of the two liquids. Then, in the one liquid, the pressure at E is the same as the pressure at B and this is the atmospheric pressure at A together with the weight of a column of liquid of unit cross section and height AB or h .

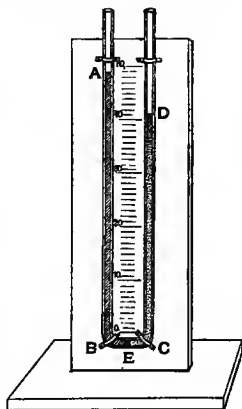


Fig. 35.

Since the cross section of the column is unity its volume is h c.cm.; its mass is $h\rho$ grammes and its weight in dynes is $h\rho g$, where g is the acceleration due to gravity.

In the second liquid the pressure at E is equal to that at C , which is equal to the atmospheric pressure at D together with the weight of a column of the second liquid of unit cross section and height h' . This weight is $h'\rho'g$ dynes.

Hence since the pressure at E is the same in the two liquids we have

$$h\rho g = h'\rho'g$$

or

$$\frac{\rho}{\rho'} = \frac{h'}{h}.$$

We have thus a method of comparing the densities of two different liquids. But the liquid in the two tubes need not be

different; the same liquid at two different temperatures may be employed, and we can thus compare its densities at two different temperatures, and so find its coefficient of dilatation.

We notice that the method just described does not depend at all on the diameter of the tubes; these may be the same as drawn in fig. (35) or they may be different, the result will be unaffected.

83. EXPERIMENT (17). *To determine the true coefficient of dilatation of a liquid.*

Take a tube $ABCD$, bent as in fig. 36. Surround the two vertical limbs AB , CD with wide jacket tubes as shewn in the figure. The ends of the tubes are closed with corks through which the vertical tubes pass. Short pieces of glass tubing are inserted in the corks, and by means of these steam or water can be passed through the jackets. Thermometers can also be inserted if necessary through the upper corks. The apparatus is held in a suitable stand with the long tubes vertical. Insert a plug of cotton wool in the horizontal tube BC and fill the two tubes with the liquid to be experimented on, say turpentine; the plug checks currents which may be set up in the tube. Fill the jacket tube surrounding CD with cold water, or better, pack it with small pieces of ice. The temperature of the liquid in CD will then be zero. Pass steam from a boiler through the jacket surrounding AB , allowing the steam to enter gradually, so as to avoid cracking the glass, at the upper end A of the jacket. The temperature of the liquid in this tube rises to 100° . When the whole has become steady it will be found that the liquid in AB stands at a considerably greater height than that in CD . Measure by

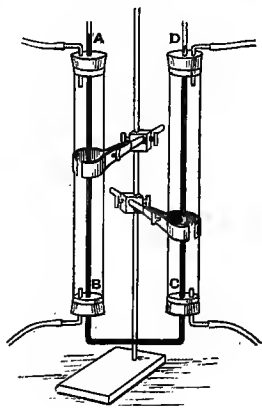


Fig. 36.

means of a vertical scale—or, if very accurate results are required, by the kathetometer—the heights AB and CD , let them be h and h_0 and let ρ and ρ_0 be the corresponding densities.

Then, since the weight of a column of unit area, height h and density ρ , is equal to the weight of a second column of the same area but of height h_0 and density ρ_0 , we have

$$h_0 \rho_0 g = h \rho g,$$

or
$$\frac{\rho_0}{\rho} = \frac{h}{h_0}.$$

But, since ρ_0 and ρ are the densities at 0° and 100° respectively, we have from the second definition of the coefficient of dilatation γ , the value

$$\begin{aligned} \gamma &= \frac{\rho_0 - \rho}{100\rho} = \frac{1}{100} \left\{ \frac{\rho_0}{\rho} - 1 \right\} \\ &= \frac{1}{100} \left(\frac{h}{h_0} - 1 \right) = \frac{h - h_0}{100h_0}. \end{aligned}$$

Thus observations of the height h_0 , and of the difference in heights $h - h_0$, give us the true coefficient of dilatation of the liquid.

If we had filled one jacket tube with water at some temperature t_0 , say, not zero, we should have proceeded in the same way, but the divisor in our result would be the difference of temperature $100 - t_0$ instead of 100° . For rough work the jacket round the cold tube may be dispensed with, and the tube assumed to be at the temperature of the air.

When the tubes were filled with turpentine the height of the column h_0 was 95.6 cm. and the difference in height was 9.8 cm.

The coefficient of expansion is thus

$$9.8/100 \times 95.6 \text{ or } .00102.$$

84. Sources of error.

(1) For a liquid with a small coefficient of expansion, such as mercury for which the value is .00018, the difference in height will not be large, and with the apparatus as described it will not be easy to measure it accurately.

(2) Some parts of both columns are outside the jackets, the temperatures of these parts are not known with any great accuracy.

(3) Heat may pass both by convection and conduction along the tube BC and the hot and cold liquids will to some extent get mixed. The plug of cotton wool serves to check the convection currents but fails to do so completely.

***85. The absolute dilatation of mercury.** The method was devised by Dulong and Petit; and the form of apparatus, used by Regnault, in which the difficulties just mentioned are overcome is shewn in fig. 37. The two tubes are connected at the top by a horizontal tube AD of narrow bore, the ends of which pass into the steam and ice jackets respectively. This tube is kept filled so that the pressures at A and D in the two columns are the same, but by making it long and narrow the transference of heat is very small. The lower connecting tube has the form shewn, the parts at F and G being vertical and close together. Thus the tubes AB and CD are completely jacketed.

The portion FG is in communication with a reservoir of compressed air in which the pressure can be varied by means of a pump.

When the two columns are at different temperatures the levels at F and G are different. The height of the hot column is very approximately the height between F and the horizontal arm AD ; that of the cold column is the height between G and the same arm. The difference in height is therefore the difference in level between F and G which can be easily measured accurately. A correction is required in consequence of the difference in temperature between the mercury in the vertical limbs FE , HG and that in AB and CD . These tubes are surrounded by a water jacket (not shewn) so that the temperature can be found accurately and the correction made.

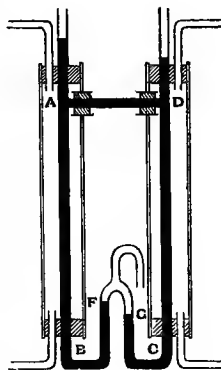


Fig. 37.

86. Measurement of the coefficient of dilatation of a solid. The coefficient of dilatation of a solid, such as the glass of a weight thermometer, may be found by a combination of the two methods of Sections 80 and 83.

For, find by means of the weight thermometer the apparent coefficient of dilatation of some liquid, say mercury, then find by the method of Section 83 its true coefficient of dilatation, the difference is the coefficient of dilatation of the glass. When once this has been found the same weight thermometer may be used to find the true coefficient of any other liquid.

***87. Dilatation at different Temperatures.** Since a degree Centigrade is defined in terms of the expansion of mercury in glass, the increase in volume of mercury in a glass tube is the same for each degree Centigrade. This however is not the case with many other liquids. Thus, for example, water increases in volume when raised from 60° to 70° by about four times the increase which takes place between 10° and 20° . The same is true though to a less extent of other liquids. Hence an alcohol thermometer is not graduated by determining two fixed points and then dividing the distance on the tube between these points into a number of equal parts. If this were done, since the increase in volume of alcohol per degree of temperature at high temperatures is greater than at low, then, on putting an alcohol and a mercury thermometer into a bath and gradually raising the temperature, the alcohol thermometer, at the lower fixed point, reads the same as the mercury instrument, but, as the temperature rises, the alcohol instrument lags behind, the reading is lower than that of the mercury thermometer, and the two agree again only when the upper fixed point is reached. For this reason an alcohol thermometer is graduated by comparison with a mercury thermometer at a large number of temperatures.

88. EXPERIMENT (18). *To observe the expansion of various liquids for a given rise of temperature at various temperatures.*

Take a thermometer tube and bulb¹ fitted with a scale of

¹ The bulb should contain from 15 to 20 c.cm. and the tube should be 25 to 30 cm. long and about 1 mm. in diameter.

millimetres such as is described in § 79 and fill it with the liquid, water suppose. Place the bulb in a bath of water with a thermometer, let the temperature be about 10° . Read the thermometer and the height of the water. Raise the bath to about 20° , by pouring in some hot water or otherwise, keep it near this temperature for some little time, until the liquid in the tube has got steady. Read again the temperature and the height of the liquid. Raise the temperature through another 10° to 30° , and so on, then, waiting in each case for the liquid in the bulb to come to the temperature of the bath; thus take a series of simultaneous readings of temperature and height of water column. It will be found that the differences between two consecutive readings get greater as the temperature rises. Enter the results in a table thus:

Temperature.	Reading.	Temperature.	Reading.
10	11	60	94.5
20	18.5	70	122.5
30	31	80	154.5
40	48.5	90	188
50	69.5	100	225.5

Thus, while the rise for the 10° from 10° to 20° is 7.5 mm., that for 10° from 60° to 70° is 28 mm. or nearly four times as much.

The results of an experiment such as this may very conveniently be shewn on a diagram by drawing two lines, OA , OB at right angles, setting off distances ON_1 , ON_2 , etc. along OA to represent temperatures, and drawing lines N_1P_1 , N_2P_2 , etc., parallel to OB to represent the height of the liquid in the tube at these temperatures. If the points P_1 , P_2 , etc. be joined by a curve-line, the expansion of the liquid at various temperatures can be graphically represented.

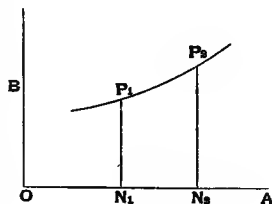


Fig. 38.

89. Dilatation of Water. In consequence of this irregularity in the expansion of water and other liquids when compared with mercury, it is not strictly accurate to speak of the coefficient of dilatation of water as the ratio of the change of volume for a rise of 1° to the original volume, without mentioning the temperature at which the change takes place. If we determine the coefficient of expansion by observing the volume at two different temperatures some way apart the result will be the average coefficient between those temperatures.

The density of water at different temperatures is most accurately determined by weighing a body, such as a piece of glass, in the water at various temperatures. If the coefficient of dilatation of the glass be known its volume at the temperatures of each of the observations is known, and since its loss of weight in water is the weight of water displaced, the weights of a known volume of water at various temperatures can be obtained.

90. Maximum Density of Water. If in the experiment described in § 88 we had started with the temperature at 0° and gradually raised it we should have found, after some calculations explained below, that water at first contracts in volume until a temperature of about 4° is reached, after which it begins to expand. Thus, a given mass of water occupies a less volume at about 4° C. than at any other temperature; its density is greatest at that temperature. A given volume of water, 1 c.cm. say, has a greater mass at 4° C. than at any other temperature; if a vessel such as a weight thermometer be completely filled with water at 4° C. and then be cooled, some of the water will overflow. We may shew this by the aid of the thermometer bulb and tube described in § 80. The following form of apparatus, in which the bulb is replaced by a coil of thin metal tubing is better, for the water will come more quickly to the temperature of the bath than is possible when a bulb is employed; besides either a narrower tube, or a larger bulb, than is required for that experiment is desirable.

EXPERIMENT (19). *To shew that water has a maximum density at about 4° C.*

Take a piece of lead tubing about 1 metre long and 1 cm. in diameter, close one end, and bend the tube into a flat spiral, turning the open end up so as to be vertical when the spiral rests on the table. Fill the tube with water, and then close the open end with an india rubber cork through which a long glass tube about 1 mm. in internal diameter passes. The water will be forced up this tube and will stand in it as at *A*. Fit a millimeter scale to the tube.

Place the flat spiral in a suitable vessel and cover it with small pieces of ice; when the column of water in the tube has become steady, note its position on the scale. Pour into the vessel containing the spiral some water which has been cooled down to freezing point, and then gradually raise the temperature of the water. It will be found that as the temperature rises the water column sinks in the tube.

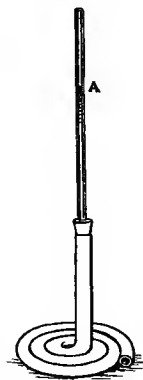


Fig. 39.

As the temperature is still further raised the column ceases to fall and finally at a temperature of about 10° begins to rise.

As the temperature is increased the lead tube expands, and, in consequence of this, even if the water expanded, provided its expansion were less than that of the tube, the water would sink; since however on raising the temperature to 10° the water rises, we see that its coefficient of dilatation is greater than that of lead at 10° , and we have seen that the coefficient increases with the temperature above 10° .

We may infer then from the experiment directly that, at low temperatures, water expands less than lead; while at a temperature of about 10° its coefficient of dilatation has become greater than that of lead. But careful measurement will permit us to deduce more exact results from the experiment. The coefficient of dilatation of lead is known, and therefore the fall which should take place in the water column owing to the expansion of the lead can be calculated. When this is done it is found that, as the temperature rises

from 0° to about 4° , the fall per degree is greater than it should be were it due to the expansion of the lead only: the water has contracted in volume from 0° to 4° ; at about 4° the fall observed is just that which would be due to the lead; while, from 4° upwards, the observed fall is not so great as it should be if the water still remained unaltered in volume. The water is expanding, but not so fast as the lead; at about 10° the water begins to rise in the tube because at that temperature its coefficient is greater than that of the lead.

If we suppose the tube to contain 100 c.cm. at 0° we shall find the following values for the apparent increase in volume at the various temperatures.

Temperature.	Observed increase in Vol.	Decrease due to Lead.	Increase of Water.
4	-.0465	.0336	-.0129
8	-.0687	.0672	-.0015
10	-.0716	.0840	.0124
15	-.0548	.1260	.0712
20	-.0065	.1680	.1615
30	+.1604	.2520	.4124

The third column gives the decrease due to the expansion of the lead, the coefficient of dilatation of which is .000084; while the fourth column gives the increase for the water, obtained by adding together the observed increase and the decrease due to the lead. The greatest decrease in the volume of the water is reached at 4° ; the volume at 8° is slightly less than at 0° , while at 10° the volume of the water has distinctly risen above its value at 0° .

91. Maximum density of Water. Hope's Experiment. The changes that take place in the density of water when near the freezing-point are shewn in the following manner by Hope's experiment.

EXPERIMENT (20). *To shew that the density of water has a maximum value at about 4° .*

In fig. 40 *ABC* is a tall metal cylinder about 30 cm. high and 10 cm. in diameter. Near the middle a circular trough some 15 cm. wide and about 10 cm. in height surrounds the cylinder. At *A* and *C* are holes closed by corks through which two delicate thermometers can be inserted into the water, which the cylinder contains; this water should be cooled down to 10° or 12° before the experiment commences. The two thermometers will then read alike. Fill the central trough *B* with a freezing mixture¹.

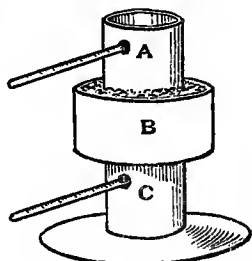


Fig. 40.

The water in the central part of the tall cylinder is thus cooled down; while this is in progress observe the thermometers. It will be found that the lower thermometer sinks gradually to about 4° . The upper thermometer remains nearly stationary at 10° or 12° , it may if the room is warm rise slightly. The water in the central part of the cylinder, as it cools, becomes denser and sinks to the bottom, lowering the temperature there; the warmer water in the upper part of the cylinder being lighter than that below remains unchanged. This continues until all the water near and below the middle has been cooled to about 4° ; as the cooling continues below this temperature the water expands, its density becomes less, and the cold water now rises instead of sinking. It will not rise to the top for the water at 10° is less dense than water at 0° . Ice however begins to be formed near the middle, and small crystals of ice rise to the surface and melting there cool the water. The lower thermometer remains at 4° and the upper thermometer falls to the same temperature. The cooling still continues, and the cooled water being now less dense than that at the top rises to the surface; hence the upper thermometer falls to 0° C. the lower one still remaining at 4° .

Thus we see that water at 4° C. is denser than at any other temperature.

¹ This, see § 110, is made by mixing well broken ice with salt.

92. Consequences of Expansion of Water. Density of Ice. The fact that water expands on becoming ice has already been alluded to, and the existence of this expansion is readily shewn in many ways. Thus ice floats on the surface of water. In cold weather pipes burst when the water they contain is frozen. We shall explain later a method of measuring this expansion and shall describe some of the consequences to which it gives rise.

The peculiar behaviour of water in expanding between the temperatures of 0°C. and 4°C. is of great importance in the economy of nature. If it were otherwise, if water steadily became denser down to freezing-point, the upper layers of water in a pond or lake would sink as they cooled, freezing would not take place till the whole mass was cooled down to 0° and then would go on rapidly through the mass. As it is the mass is cooled down rapidly by convection currents to 4° . Below that temperature the cooled water above is less dense than the warmer water below; it therefore remains at the top, the cooling below 4°C. takes place by conduction only, and this is a much slower process than the transference of heat by the actual currents of the water. Thus the greater part of the water in the pond remains at 4° while a coat of ice is formed on the surface.

93. Corrections to a barometer reading for expansion. Some of the practical results of the expansion of solids have already been noticed; one other important measurement in which corrections have to be made for expansion should be mentioned. The pressure of a gas is often measured in terms of the height of a column of some liquid; thus the height of the barometer measures the pressure of the atmosphere, but the height of a column of liquid required to balance a given pressure will vary with the temperature of the liquid; while the length of the scale used to measure the height will also change. For both these reasons we need to know the temperature at which the height is measured. In considering the correction required in consequence of the expansion of the liquid—mercury suppose,—we must remember that the pressure is measured by the weight of a column of

mercury of unit area and of the given height. The weight of such a column is proportional to the density of the mercury, and the density of the mercury depends on its absolute coefficient of dilatation.

Now h , the observed height, needs correction because the scale is not accurate at the temperature of the observation but at some other; let us suppose it correct at freezing-point, and let H be the real height of the column at the temperature of the observation; let α be the coefficient of linear expansion of the material of the scale. Then since the scale is, at t° each centimetre is really $1 + \alpha t$ cm. in length. The length of the column is apparently h cm., hence its real length is

$$h(1 + \alpha t) \text{ cm.},$$

or

$$H = h(1 + \alpha t).$$

This height H now requires correcting because the mercury is at t° instead of at the standard temperature zero. Let H_0 be the corrected height, ρ the density of the mercury at t° , ρ_0 at zero, γ the coefficient of dilatation of the mercury. We know that

$$\rho_0 = \rho(1 + \gamma t).$$

Now the column of height H_0 is to have the same weight per unit area as the column of height H .

Thus we must have

$$\rho H = \rho_0 H_0 = \rho H_0(1 + \gamma t).$$

$$\text{Therefore} \quad H_0 = \frac{H}{1 + \gamma t} = H(1 - \gamma t),$$

if we neglect terms in $\gamma^2 t^2$, etc.

Hence finally introducing the value of H in terms of h the observed height,

$$H_0 = h(1 + \alpha t)(1 - \gamma t).$$

Thus we find the corrected height H_0 at freezing-point in terms of the observed height h .

¹ See § 64, p. 64.

The value thus found may be simplified; thus on multiplying out we have,

$$H_0 = h(1 + at - \gamma t - a\gamma t^2).$$

Now $a\gamma t^2$ is very small compared with at and γt ; hence

$$H_0 = h\{1 - (\gamma - a)t\}.$$

The above argument has made it clear that in this problem we are concerned with the absolute dilatation of the mercury, not with its coefficient relative to glass. We may see in another way that this is so, if we remember that the size of a barometer tube does not affect the height of the column; if the temperature of the glass tube could be raised without heating the mercury the height of the column would not change: more mercury would rise from the cistern to fill the additional space caused by the expansion of the tube; if now the temperature of the mercury be raised, the whole of the contents of the tube expand, and the amount of expansion depends on the absolute dilatation of the mercury.

94. Values of coefficients of dilatation. We will close this account of the dilatation of liquids with a table of approximate coefficients of dilatation.

	Mean coefficient of dilatation.	Between
Alcohol	·00109	0—120
Benzine	·00138	11—80
Ether	·00210	0—33
Mercury	·00018	0—100
Turpentine	·00105	—9—106

EXAMPLES.

EXPANSION OF LIQUIDS.

1. A glass bulb with a uniform fine stem weighs 10 grms. when empty, 117·3 grms. when the bulb only is full of mercury, and 119·7 grms. when a length of 10·4 cms. of the stem is also filled with mercury: calculate the relative coefficient of expansion of a liquid which, when placed in the same bulb, and warmed from 0°C. to 28°C. , expands through the length from 10·4 to 12·9 cm. of the stem. The density of mercury is 13·6 grammes per c.cm.

2. What is meant by the coefficient of expansion of a liquid?

If the coefficient of expansion of mercury in glass be $\frac{1}{8500}$, what mass of mercury will overflow from a weight thermometer containing 360 grammes of mercury at 0°C . when the temperature is raised to 98°C . ?

3. Describe the process of determining the coefficient of expansion of a liquid like alcohol or paraffin, remembering the necessary preliminary determination of the expansibility of the glass vessel employed.

4. Distinguish between the coefficient of apparent expansion and the coefficient of absolute expansion of a liquid. By what method has the coefficient of absolute expansion of mercury been determined?

5. The weight of mercury in a weight thermometer at 0°C . is found to be 25 grammes, and at 100°C . 24.630 grammes; find the coefficient of expansion of mercury in glass. If the absolute coefficient of expansion of mercury be .000189 per 1°C ., find the coefficient of linear expansion of glass.

6. Explain why in reading a barometer it is necessary to correct the reading for the temperature of the mercury. A barometer with a glass scale reads 755 mm. at 18°C .; find the reading at 0°C . The apparent coefficient of expansion of mercury in glass is .000155, and the coefficient of linear expansion of glass is .0000089.

7. A barometer has a brass scale which is correct at 60°Fahr. : the barometer reads 754 at 40°Fahr. ; correct the reading of the barometer to 32°Fahr.

[The coefficient of linear expansion of brass is .00001; the coefficient of cubical expansion of mercury is .0001 per 1°Fahr.]

8. The coefficient of absolute expansion of mercury is .000182. A weight thermometer of glass contains 10 grammes of mercury at 0° and 9.843 grammes at 100° ; find the coefficient of expansion of the glass.

9. A glass bulb contains 800 gms. of mercury at 0°C . It is heated to $99^{\circ}.5\text{C}$., and 12 gms. of mercury are expelled. Assuming the mean coefficient of expansion of mercury between 0° and 100°C . to be .0001816, find that of the glass.

10. The coefficient of cubical expansion of mercury is .000180, and of brass .000060, per 1°C . Find the atmospheric pressure in inches of mercury at 0°C ., when a barometer with a brass scale (correct at 62°F .) reads 30 in. at a temp. of 50°F .

CHAPTER VII.

DILATATION OF GASES.

95. Boyle's Law. The volumes of most bodies can be changed by change of pressure. For solids and liquids however this change is extremely small, and, therefore, in dealing with the dilatation of such bodies due to rise of temperature it has not been necessary to notice those changes in volume which may be produced by variation of pressure.

A gas, on the other hand, alters in volume considerably for small changes of pressure, even though the temperature remain constant, and we require to investigate first the law which regulates this change. This law, called Boyle's Law, was first enunciated by the Hon. Robert Boyle in 1662.

Boyle's Law. *The pressure of a given mass of gas at constant temperature is inversely proportional to its volume.*

EXPERIMENT (21). *To verify Boyle's Law.*

In fig. 41 *AB*, *CD* are two glass tubes connected by stout india-rubber tubing and fixed to a vertical stand. *AB* is closed at its upper end and may be 50 cm. long and .5 cm. in diameter; *CD* is a wider tube and is open at the top. A vertical scale parallel to the tubes is

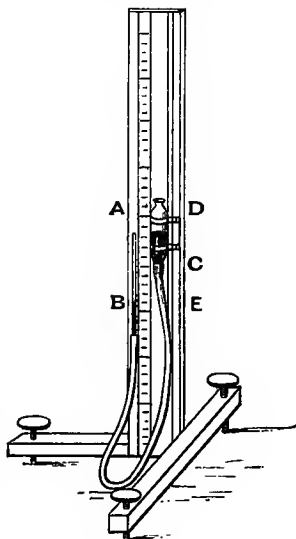


Fig. 41.

attached to the stand, and CD can slide up and down this scale. The india-rubber tubing and the lower parts of the glass tube contain mercury. The upper part of the tube AB is filled with dry air, which constitutes the given mass of air on which the experiment is to be made. The volume of this air is proportional to the length, AB , of the tube which it occupies, and this length can be read off directly on the scale. To find the pressure of the air, let the horizontal line through B meet the mercury in the moveable tube at E , and let D be the top of this mercury column.

Then the pressure at B is equal to the pressure at E , and this is equal to the pressure of the atmosphere at D together with the weight of a column of mercury of unit area and height DE . Thus, if b cm. be the height of the barometer, the pressure at B is measured by a column of mercury of height $b + DE$.

Now, according to Boyle's Law, if the temperature is constant the pressure is inversely proportional to the volume. Hence if the pressure and the volume of the gas in AB be multiplied together the product obtained will be always the same.

Raise or lower the sliding tube until the mercury stands at the same level in the two tubes.

Read on the scale the level of the top of the tube AB and the position of the mercury in the tube; the difference will be proportional to the volume of the air. Since the mercury in the two tubes is at the same level the pressure of the enclosed air is the atmospheric pressure. Observe the height of the barometer, let it be b cm.; the pressure of the enclosed air is measured by a mercury column b cm. in height. Raise the sliding tube. The mercury in the closed column also rises but not so fast; the volume of the enclosed air is reduced, but its pressure is increased, being now measured by the height of the barometer together with the column of mercury between the two levels. Continue to raise the sliding tube until the mercury in the closed tube reaches a point B' , half-way between A and B ; let the level of the mercury in the open tube when this is the case be at D' . The volume of the enclosed air is now half what it was. Its pressure is

measured by $b + D'B'$. Read the levels at D' and B' , it will be found that $D'B'$ is equal to b , the height of the barometer; hence the pressure of the enclosed air is now measured by $2b$; that is, it is twice what it was originally. The volume has been halved, the pressure has been doubled. Thus Boyle's Law has been verified for this case.

The verification may be made more complete by taking readings of the volume and of the pressure in a number of other positions of the sliding tube.

If B_1, D_1 be corresponding levels of the two mercury columns, the volume of air is proportional to AB_1 , the pressure to $b + B_1D_1$. Set down in two parallel columns the values of AB_1 , and of $b + B_1D_1$, and in a third column the numbers obtained by multiplying the corresponding values together. It will be found that these products are constant within the limits of experimental error.

The results of the experiments may be entered in a table thus:

Volume.	Pressure.	PV.
50	76	3800
40	$76 + 18.8$	3792
30	$76 + 50.5$	3795
20	$76 + 113$	3780

The same apparatus may be used for pressures less than that due to the atmosphere by lowering the position of the sliding tube until D is below B ; in this case the pressure is given by $b - BD$.

96. Deductions from Boyle's Law. Boyle's Law may be expressed in symbols in various ways. Thus if p be the pressure, v the volume of a given mass of gas; then, since the pressure is inversely proportional to the volume, we have the result that the ratio of p to $1/v$ is constant; denoting this constant by k we find

$$\frac{p}{\frac{1}{v}} = k.$$

Therefore
$$p = \frac{k}{v},$$

or
$$pv = k;$$

when we say that k is a constant we mean that it does not change when the pressure and volume are changed, if the temperature and mass of the gas are not varied. When a gas is allowed to expand under the condition that the temperature does not alter the expansion is said to be isothermal. If corresponding values of the pressure and volume are plotted the curve formed is said to be an isothermal curve.

Or again, if the volume v becomes v' , and in consequence the pressure p is changed to p' , since the product of the pressure and volume does not alter we have $pv = p'v'$.

Again, the volume of a given mass of gas is inversely proportional to its density; since, therefore, the volume is inversely proportional to the pressure, we see that the pressure of a gas is proportional to its density; or if ρ be the density, the ratio of p to ρ is a constant. We may write this $p = k\rho$, where k is a constant.

Examples involving Boyle's Law may be worked in various ways. Thus we may use the formula directly as in the following.

(1) *The volume of a mass of gas at 740 mm. pressure is 1250 c.cm., find its volume at 760 mm.*

Let v be the new volume, then from the formula $pv = p'v'$

$$v \times 760 = 1250 \times 740,$$

$$v = 1250 \times 74/76 = 1217.1 \text{ c.cm.}$$

Or we may preferably put the argument in full thus:

Volume at pressure 740 is 1250 c.cm.

Volume at pressure 1 is 1250×740 c.cm.

Volume at pressure 760 is $\frac{1250 \times 740}{760}$ c.cm.

It is of course by no means necessary to measure the pressure in terms of the height of a column of mercury. Thus,

(2) *A bubble of gas 100 c.mm. in volume is formed at a depth of 100 metres in water, find its volume when it reaches the surface, the height of the barometer being 76 cm.*

Since the density of mercury is 13.59 grammes per c.cm., the height of the water barometer is 76×13.59 cm., and this is very approximately 10.34 metres.

Thus the pressure at the surface is measured by a column of water 10·34 metres high, that at 100 metres by a column 110·34 metres.

Hence the new volume = $\frac{100 \times 110 \cdot 34}{10 \cdot 34}$ or 1067 c.mm. approximately.

(3) The mass of a litre of air at 760 mm. pressure is 1290 grammes. Find the mass of 1 cubic metre of air at a pressure of 1·9 mm.

The volume of the given mass of air at a pressure of 1·9 mm. is 1000000 c.cm.

Therefore the volume at pressure of 76 cm. is

$$\frac{10000 \times 19}{76} \text{ or } 2500 \text{ c.cm.}$$

The mass of 1 c.cm. air is ·001293 grammes.

Therefore the mass of 2500 c.cm. is $2500 \times \cdot 001293$ grammes, and this is 3·24 grammes.

97. Variations from Boyle's Law. More exact experiments have shewn that Boyle's Law is not absolutely true, though for the so-called permanent gases, oxygen, hydrogen, nitrogen and others, it holds very nearly; other gases, such as carbonic acid, which can be condensed to a liquid at ordinary temperatures by the application of a not very large pressure (see § 119), deviate more from the law.

98. Dilatation of gases by heat. The law of the dilatation of gases for rise of temperature was first stated by Charles.

Charles' Law. *The volume of a given mass of any gas, at constant pressure, increases for each rise of temperature of 1° C. by a constant fraction (about 1/273) of its volume at 0° C.*

Thus if the volume at 0° be v_0 c.cm. the increase for each rise of 1° is $v_0/273$, therefore for t° the increase in volume is $v_0 t/273$, hence if v c.cm. be the volume at t°

$$v = v_0 + \frac{v_0 t}{273} = v_0 \left(1 + \frac{t}{273} \right).$$

The formula resembles those we have already arrived at for solids and liquids; some important points of difference should be noted.

(1) It is stated definitely that the pressure is to be kept constant; the formula is not true unless this is the case.

(2) The increase of volume is defined as a fraction ($1/273$ or $\cdot00366$) of the volume at 0°C . This point has been already noticed; we have seen that the dilatation of most solids is so small that we may measure the increase in volume at any temperature as a fraction of the volume at that temperature without error; and also that if we adopt the same method for most liquids, the error will not be large. For gases however the dilatation per degree rise of temperature is greater, and the results will not be correct unless it be defined as a fraction of the volume at 0° .

(3) The coefficient of dilatation ($1/273$ or $\cdot00366$) is very approximately the same for all gases; this approximation is more close the nearer the gases obey Boyle's law.

Thus we have the following values of the coefficients.

Gas.	Coefficient.	Gas.	Coefficient.
Hydrogen	$\cdot00366$	Carbonic Acid	$\cdot00371$
Air	$\cdot00367$	Nitrous Oxide	$\cdot00372$
Nitrogen	$\cdot00367$	Sulphurous Acid	$\cdot00390$
Carbonic Oxide	$\cdot00367$	Cyanogen	$\cdot00388$

99. EXPERIMENT (22). *To shew that equal volumes of different gases expand by the same amount for a given rise of temperature.*

Take a flask containing say 50 c.cm., close it with a cork through which passes a glass tube bent twice at right angles as BC in fig. 42, let the end C of the tube pass through a cork into a test tube about 20 c.cm. in volume. Pass a second tube DE through the cork; let one end, D , reach to the bottom of the test tube, while the other is bent over as at E .

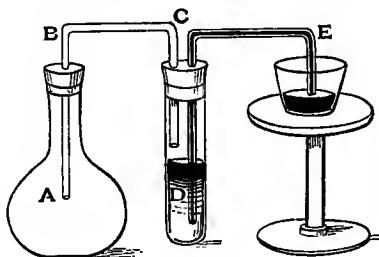


Fig. 42.

Fill the test tube with dry mercury. Prepare two of these pieces of apparatus. Fill one of the 50 c.cm. flasks with air, the other with some other gas, coal-gas, hydrogen or oxygen. Place the two flasks and test tubes side by side in a water bath, in such a way that the end *E* of the second tube projects out over the edge, and place a weighed beaker to catch the mercury which will be expelled from the test tubes as the temperature rises. Take the temperature of the bath, let it be t_0° . We have then in the two flasks equal volumes (50 c.cm.) of air and hydrogen or coal-gas respectively. Now raise the temperature of the bath by pouring in hot water or passing steam through. In each apparatus the gas expands into the test tube, expelling some of the mercury into the beaker; let the new temperature be t_2° . Weigh the mercury collected in each of the two beakers; it will be found that the weights are the same. The volume of this mercury measures the increase in volume of the 50 c.cm. of air or hydrogen. Thus for a given rise of temperature¹ equal volumes of these two gases have expanded by equal amounts.

100. EXPERIMENT (23). *To determine the coefficient of dilatation of a gas at constant pressure.*

Take a piece of glass tubing *AB*, fig. 43, about 1 mm. in bore and not less than 20 cm. long. Pass dry air through it by means of an aspirator or pump, and then close one end with

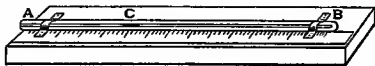


Fig. 43.

a blowpipe flame. Warm the tube so as to expel some of the air, then dip the open end under dry mercury, and so suck up, as the air cools, a small pellet of mercury. Place the tube vertically with its open end upwards and leave it to cool.

¹ In this experiment the pressure has not been kept quite constant, the change in pressure however has been the same in the two and affects them equally. By determining the volume of mercury expelled and allowing for the change in pressure we may calculate the coefficient of dilatation.

The amount of air expelled should be such that when the tube comes to the temperature of the room the pellet *C* may stand at a height of some 14 or 15 cm. above the bottom. Attach to the tube¹ a millimetre scale graduated from the bottom.

The tube is thus an air thermometer such as is described in § 23 but without a bulb.

Place it in melting ice with the open end upwards; the enclosed air contracts and the mercury pellet sinks in the tube; when it has become steady read on the scale the position of its lower end; let the reading be a mm. If the bore of the tube be uniform, the volume of the enclosed air at 0° is proportional to a .

Remove the tube from the ice and place it in boiling water; the enclosed air expands: when the pellet has become steady read its position on the scale; let it be b mm. from the bottom, the volume of the air at 100° is proportional to b . Thus the ratio of the increase of volume for 100° to the volume at 0° is $(b-a)/a$. Dividing this by 100 we get as the mean coefficient of dilatation the value

$$\frac{b-a}{100a}.$$

Example. In an air thermometer as described above the pellet stood at 145 mm. in ice and at 198 mm. in boiling water.

Thus the coefficient of dilatation is

$$(198-145)/145 \times 100 \text{ or } 53/145 \times 100,$$

and this comes to be about $1/274$ or $\cdot 00365$.

101. Graduation of an Air Thermometer. Absolute Temperature. Let us now suppose that we wish to graduate an air thermometer such as that described in Experiment 23. Make a mark on the tube to indicate the positions of the bottom of the pellet in melting ice and in boiling water, or better in the steam issuing from boiling water, respectively. Divide the distance between these graduations into 100 equal parts, and continue the gradua-

¹ It is convenient to use for the experiment an old thermometer tube on which the graduations have been engraved.

tions down to the bottom of the tube. It will be found that between the freezing-point and the bottom of the tube there are 273 divisions¹; so that if we agree to call freezing-point 0° and boiling-point 100°, we must call the bottom of the tube -273°. Or again if we call the bottom of the tube 0°, freezing-point will be 273° and boiling-point 273° + 100° or 373°.

Now we have seen that the air expands through 100 of these divisions for the rise of temperature from freezing to boiling point or through 100° C.

If now we compare our air thermometer with a Centigrade thermometer by placing them in a bath and raising the temperature, we find that, practically, they agree all the way up the scale. When the air column has expanded through ten divisions the mercury thermometer reads 10° and so on.

The expansion of air at constant pressure—and unless the barometer varies during the experiment the pressure is constant—is regular when measured in terms of the mercury thermometer; each of the divisions on the air thermometer corresponds to the increase or decrease of volume produced by a change of temperature of 1° Centigrade. The scale of the mercury thermometer agrees very closely with that of the air thermometer².

If now the temperature be lowered below freezing-point, the mercury pellet sinks through 1 division for each fall of temperature of 1°, and, if we could suppose the gas to continue to contract according to the same law for any fall of temperature however great, on reducing the temperature to -273° C. below freezing-point the mercury pellet would have reached the bottom of the tube and the volume of the air would have become zero.

This temperature is called the absolute zero of the air thermometer, and the temperature measured from absolute

¹ This is clear from the formula

$$(b - a)/100a = \text{coefficient of dilatation} = 1/273.$$

Hence if $b - a = 100$ we have $a = 273$.

² Careful research has shewn that this agreement is not absolute and depends greatly on the glass used in the mercury thermometer.

zero is called absolute temperature. Each degree on the air thermometer is equal to one degree Centigrade, and we may say that absolute zero is -273° C. below freezing-point, or that freezing-point is 273° Absolute above zero.

Again, since t° C. means t° above freezing-point, and freezing-point is 273° above absolute zero, we see that t° C. corresponds to $273+t^{\circ}$ absolute; that is, to find the measure of a temperature on the absolute scale, we have to add 273° C. to the Centigrade temperature.

Again, temperature on the absolute scale is measured by the height of the mercury pellet above the bottom of the thermometer tube, but the volume of the air column is also proportional to this same height; thus we get the result that *The volume of a given mass of gas at constant pressure is proportional to its absolute temperature.*

This is the form of the law connecting the volume and temperature of a gas which is most useful for numerical calculations.

We can obtain the above results more rapidly from the formula thus:

$$\text{We have} \quad v = v_0 \left(1 + \frac{t}{273} \right).$$

Let t be the temperature in Centigrade degrees at which the volume is zero.

$$\begin{aligned} \text{Then} \quad 0 &= v_0 \left(1 + \frac{t}{273} \right). \\ \therefore t &= -273^{\circ}. \end{aligned}$$

We call this the absolute zero of the air thermometer, and say that absolute temperature is temperature reckoned from absolute zero.

Let T be the absolute temperature corresponding to t° C. and put T_0 = absolute temperature of freezing-point = 273° .

$$\text{Then} \quad T = t + 273 = t + T_0.$$

$$\text{Also} \quad v = v_0 \frac{(273 + t)}{273} = \frac{v_0 T}{T_0}.$$

$$\therefore \frac{v}{T} = \frac{v_0}{T_0}.$$

That is, the volume of a mass of gas at constant pressure is proportional to its absolute temperature.

We have worked hitherto in degrees Centigrade; but since a degree Fahrenheit is $\frac{5}{9}$ of a degree Centigrade and the dilatation of a gas is $\frac{1}{273}$ per degree Centigrade, its dilatation per degree Fahrenheit is $\frac{5}{9}$ of $\frac{1}{273}$ or about $\frac{1}{491}$. Thus the absolute zero on Fahrenheit's scale is 491° Fahrenheit below freezing-point, and since zero Fahrenheit is 32° Fahrenheit below freezing-point the absolute zero is $491^\circ - 32^\circ$ or 459° Fahrenheit below zero Fahrenheit.

Again, since the coefficients of dilatation of all permanent gases are nearly the same, the absolute zero of any gas thermometer is approximately the same temperature.

In the above the fraction $1/273$ has been used throughout as the coefficient of dilatation of a gas. The results might have been made more general by the employment of a symbol α for this quantity. In this case we should have the equation

$$v = v_0(1 + \alpha t),$$

and the temperature at which v is zero will be $-1/\alpha$. Thus if α is the coefficient of expansion of any gas the temperature of the absolute zero of that gas is $-1/\alpha$. Let us put

$$T_0 = \frac{1}{\alpha}, \quad T = T_0 + t = \frac{1}{\alpha} + t.$$

Then

$$v = v_0(1 + \alpha t) = v_0 \alpha \left(\frac{1}{\alpha} + t \right)$$

$$= \frac{v_0}{T_0} \cdot T.$$

Thus

$$\frac{v}{T} = \frac{v_0}{T_0},$$

or the volume is proportional to the absolute temperature.

Lord Kelvin has shewn how to construct a scale of temperature known as "Thomson's Absolute Scale" which is independent of the substance in the thermometer, be it gas, mercury or anything else. He has further proved by the experiments of Joule and himself that this absolute scale is very nearly identical with that of the air thermometer. Hence the terms "absolute" scale, "absolute" temperature, etc. which apply really in strictness only to Thomson's scale are used in connection with the air thermometer. (See Maxwell's *Heat*, Chapter XIII.)

Examples. (1) *A litre of hydrogen at 15° C. is heated at constant pressure to 100° C., find its volume.*

The absolute temperatures are $15 + 273$ or 288° and $100 + 273$ or 373° . Hence if v is volume

$$v = \frac{373}{288} \times 1000 \text{ c.cm.} = 1295 \text{ c.cm.};$$

or we may express the argument in full thus,

Volume at 288° absolute is 1000 c.cm.

Volume at 1° absolute is $\frac{1000}{288}$ c.cm.

Volume at 373° absolute is $\frac{373}{288} \times 1000$ c.cm.

\therefore required volume is 1295 c.cm.

(2) *The temperature of a litre of gas is 27° C. At what temperature will the volume have increased by 500 c.cm.?*

$$27^\circ \text{ C.} = 273 + 27 = 300^\circ \text{ Abs.}$$

Hence if T is the required temperature

$$\frac{T}{1500} = \frac{300}{1000},$$

$$T = 450^\circ.$$

\therefore the Centigrade temperature is

$$450^\circ - 273^\circ \text{ or } 177^\circ \text{ C.}$$

102. Law connecting Pressure, Volume and Temperature of a Gas. By combining Boyle's Law and Charles' Law we can obtain a relation between the pressure, volume and temperature of a gas when all are allowed to vary in the following way.

Consider a mass of gas; let p , v and T be the pressure, volume and temperature of the gas, the temperature being measured from absolute zero.

Let the pressure be changed to p_1 and the temperature to T_1 , and in consequence let the volume become v_1 ; we require to find v_1 . We may suppose the two changes to take place separately and reason thus; during the first change the temperature remains constant and Boyle's Law holds, during the second the pressure is constant and Charles' Law is true.

Thus we get

Volume at pressure p , temperature T , is v .

Volume at pressure 1, temperature T , is pv .

Volume at pressure p_1 , temperature T , is $\frac{pv}{p_1}$.

Volume at pressure p_1 , temperature 1, is $\frac{pv}{Tp_1}$.

Volume at pressure p_1 , temperature T_1 , is $\frac{pvT_1}{Tp_1}$.

Therefore
$$v_1 = \frac{pvT_1}{Tp_1},$$

or
$$\frac{p_1 v_1}{T_1} = \frac{pv}{T}.$$

The reasoning may be made clearer by a diagram in the following way.

Let the air be contained in a cylinder, fig. 44, with a piston, and let A be the position originally. When the pressure is changed to p_1 , the temperature remaining constant, let the piston come to A' and let v' be the new volume. And finally, when the temperature is changed to T_1 , the pressure remaining p_1 , let A_1 be the position of the piston.

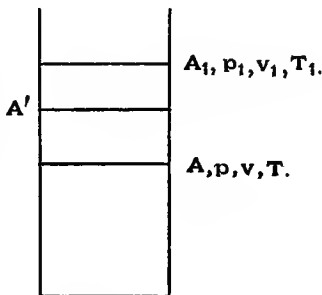


Fig. 44.

Then for the first change from p to p_1 , T being constant, Boyle's Law holds and

$$v' = \frac{pv}{p_1}.$$

For the second change from T to T_1 , p_1 being constant, Charles' Law holds, and

$$v' = \frac{Tv_1}{T_1}.$$

Therefore
$$\frac{pv}{p_1} = \frac{T v_1}{T_1},$$

or
$$\frac{pv}{T} = \frac{p_1 v_1}{T_1}.$$

A short algebraic proof can be given thus. Since v varies inversely as p when T is constant and v varies as T when p is constant we know from Algebra that v varies as T and inversely as p when p and T both vary. Hence pv/T is constant.

103. Change of Pressure at Constant Volume.

A case of the above law of special importance arises when the changes of pressure and temperature are such that the volume remains constant. We are then to have $v = v_1$; putting this value in the formula

we find
$$\frac{pv}{T} = \frac{p_1 v}{T_1},$$

or
$$\frac{p}{T} = \frac{p_1}{T_1}.$$

In other words, the pressure of a gas when the volume is kept constant, is proportional to its temperature reckoned from the absolute zero of the air thermometer.

Again, let the temperature from which the experiment is started be that of the freezing-point, so that $p_1 = p_0$, the pressure at the freezing-point, and $T_1 = T_0$, the absolute temperature of freezing-point; on turning to Centigrade temperature we have

$$\begin{aligned} T_0 &= 273^\circ, & T &= 273 + t, \\ p &= \frac{p_0 T}{T_0} = p_0 \frac{(273 + t)}{273} \\ &= p_0 \left(1 + \frac{1}{273} t \right). \end{aligned}$$

And this is an equation of exactly the same form as that found for the volume of a gas at constant pressure in § 101.

Thus we may say that *The pressure of a gas at constant volume increases by a constant fraction ($\frac{1}{273}$) of the pressure at the freezing-point for each rise of temperature of 1° Centigrade.*

This law has been deduced by an application of Boyle's Law and Charles' Law. Since it has been shewn that gases do not obey Boyle's Law exactly, the result, in the form given, is not absolutely true. Careful experiments shew that the fraction which multiplies t in the two formulæ is slightly different. The difference however is too small to concern us here.

The law which has just been enunciated can be verified by a modification of Balfour Stewart's constant volume air thermometer.

104. EXPERIMENT (24). *To prove that the pressure of a mass of gas at constant volume increases by $\frac{1}{273}$ of the pressure at 0°C. for each rise of temperature of 1° , and to find the coefficient of increase of pressure of a gas at constant volume.*

The apparatus required is similar to that described in § 95 for verifying Boyle's Law. The closed tube AB is removed, and is replaced by a tube shewn in fig. 45 at AB , bent twice at right angles, and terminating in a bulb B , some 5 or 6 cm. in diameter. The horizontal tube and part of the vertical tube near A are of narrow bore; the lower part of the vertical tube where it joins the india-rubber tubing is wider. The bulb is filled with clean dry air, and a mark is made at A , either on the glass, or on the stand behind the glass. In performing the experiment the mercury is always brought up to this mark by raising or lowering the sliding tube CD . The volume of the air in the bulb and tube is thus kept constant.

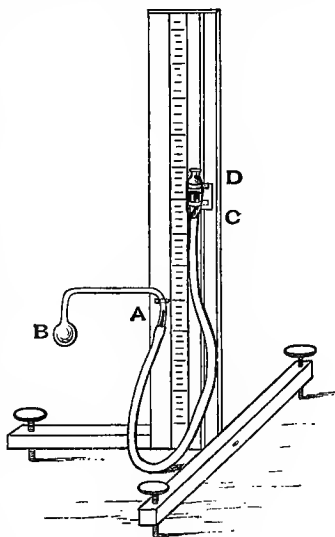


Fig. 45.

Now place the bulb in melting ice; as the air cools its pressure becomes less and the mercury tends to rise above A , but, by adjusting the sliding tube, it can be kept constantly at A . When the column has become steady, read the height of the mercury in the sliding tube CD above A . Add this height to the height of the barometer and thus find the pressure p_0 under which the air at 0° has the given volume. If D be below A the distance AD must be subtracted from the height of the barometer to give the pressure. Remove the bulb from the ice and place it in a vessel of warm water, which can be made to boil, or in steam. The air expands and the mercury is driven down below A , but by raising the sliding tube it can be brought back to A . Do this, and when the temperature has become steady, read the height of the mercury in the sliding tube above A . Add this to the height of the barometer, and so find p the pressure of the air at 100° . Subtract p_0 from p ; divide the result by p_0 and by 100, and thus find the mean coefficient of increase of pressure between 0° and 100° .

To prove that the increase of pressure is uniform, let the bath cool slowly, keeping the level of the mercury steady at the mark A , and as it cools past each tenth degree, 90° , 80° , etc. read the pressure; if the cooling take place sufficiently slowly, so that the water of the bath and the air in the bulb may be at the same temperature, the differences between these pressure readings, which give the fall of pressure for each 10° , will be the same. These observations might have been made as the temperature was rising. It may not be possible to take the observations exactly at each tenth degree, in this case take a series of simultaneous readings of pressure and temperature, then form a series of corresponding differences of pressure and of temperature, by subtracting each pressure reading from the previous one, and similarly for the temperatures. Divide the pressure differences by the corresponding differences of temperature, and so find a series of values of the increase of pressure per degree of temperature at different parts of the scale. These values should all be equal. To find the coefficient of increase of pressure they must be divided by the pressure at 0° , the quotient will be approximately $\frac{1}{273}$ or .00366.

Examples. These may be worked either directly from the formula or by the unitary method already employed in § 96.

Thus 500 c.cm. of air at 22° C. and a pressure of 730 mm. are cooled to the standard temperature 0° C. and a standard pressure 760 mm.; find the volume.

The absolute temperatures are

$$273 + 22 \text{ or } 295^{\circ} \text{ and } 273^{\circ}.$$

$$\begin{aligned} \text{Thus} \quad v &= \frac{730 \times 500}{295} \times \frac{273}{760} \\ &= 444.5 \text{ c.cm.,} \end{aligned}$$

or otherwise

Volume at 295° and 730 is 500 c.cm.

Volume at 295 and 1 is 500×730 c.cm.

Volume at 1 and 1 is $\frac{500 \times 730}{295}$ c.cm.

Volume at 273 and 1 is $\frac{500 \times 730 \times 273}{295}$ c.cm.

Volume at 273 and 760 is $\frac{500 \times 730 \times 273}{295 \times 760}$ c.cm.

Hence new volume = 444.5 c.cm.

***105. Forms of Air Thermometers.** The instrument which has just been described is the constant volume air thermometer, in it the temperature is measured by means of the increase in pressure of the air. The instrument described in § 100 is with some small modifications a constant pressure air thermometer, the temperature is there measured by the expansion of the air at constant pressure.

Other forms of air thermometers are sometimes employed; some of these have already been described § 23. The following is one which is useful for high temperatures. A bulb is taken with a narrow tube attached. This is drawn out to a fine point. The bulb is raised to the temperature which we require to measure and the end of the tube sealed off with a blowpipe; the air in the bulb fills the bulb at atmospheric pressure, and at the temperature T to which it was heated: if we can find the volume of the same mass of air at some other temperature, we can find T . Weigh the bulb. Place it with

the tube downwards under water and break off the end of the tube with a sharp file. The water enters the bulb. Adjust it till the level of the water in the bulb is the same as that of the water outside. Close the end of the tube with the finger, invert the bulb and lift it out of the water. Dry the outside and weigh it again. The bulb is not filled with the water and the volume occupied by air is the volume, at the temperature of the bath T_1° , of the air which at T° filled the bulb.

We wish to find this volume. Fill the bulb with water completely and weigh again. The difference between the last two weighings gives the mass of water which fills the space whose volume we wish to find; if the mass be found in grammes, we obtain at once the volume we require in c.cm.; let this volume be v_1 . The difference between the last weighing and the mass of the empty bulb will be the mass of water which fills the bulb, and this gives the volume v of the bulb.

Thus a mass of air which at temperature T fills the volume v has at temperature T_1 the volume v_1 .

$$\text{Therefore} \quad T = T_1 v / v_1.$$

The temperatures are of course absolute temperatures.

Example. The following observations were made in an experiment as just described:

Mass of empty bulb 14.53 gr.

Temperature of bath 12°C .

Mass of partly filled bulb 43.77 gr.

Mass of bulb when full 65.64 gr.

$$\text{Hence} \quad T_1 = 273 + 12 = 290^\circ \text{ absolute.}$$

$$v_1 = 43.77 - 14.53 = 29.24 \text{ c.cm.}$$

$$v = 65.64 - 14.53 = 51.11 \text{ c.cm.}$$

$$\text{Whence} \quad T = \frac{51.11 \times 290}{29.24} = 506^\circ.8.$$

Hence the temperature required in Centigrade degrees is
 $506^\circ.8 - 273^\circ$ or $233^\circ.8$.

In the above calculations we have neglected the effect of the expansion of the glass bulbs and tubes which contain the gas. Since the coefficient of dilatation of glass is very small compared with that of air, .000023 as against .00366, the error is too small to be perceptible without more delicate apparatus.

EXAMPLES.

EXPANSION OF GASES.

1. State Boyle's Law and express the results on a diagram in which the pressure is represented by vertical lines, the volume by horizontal lines. If the gas be more compressible than is given by Boyle's Law, how does the curve differ from the above?

2. Enunciate the laws of Boyle and Charles. If a gas has a volume of 45 litres at a pressure of 760 mm. and temperature $27^{\circ}\text{C}.$, at what pressure will its volume be 30 litres when the temperature is $77^{\circ}\text{C}.$?

3. Explain accurately what is meant by the statement that the coefficient of expansion of air is $1/273$. The volume of a certain quantity of air at $50^{\circ}\text{C}.$ is 500 cubic inches. Assuming no change of pressure to take place, determine its volumes at $-50^{\circ}\text{C}.$ and at $100^{\circ}\text{C}.$ respectively.

4. State the effect on the volume of a given mass of air, of altering its temperature without altering its pressure; also the effect on its pressure of altering its temperature without altering its volume.

5. A mass of air occupies 25 cubic feet at a temperature of $15^{\circ}\text{C}.$ and a pressure of 15 lbs. per square inch. What will be its volume at $100^{\circ}\text{C}.$ and a pressure of 25 lbs.?

6. What will be the volume of a mass of air measuring 10 c. feet at $0^{\circ}\text{C}.$ if the temperature be raised to $273^{\circ}\text{C}.$ and the pressure doubled?

7. State the gaseous laws and shew how it follows from them that for a gas at pressure p , volume v , and absolute temperature T , pv is proportional to T . Explain carefully what you mean by absolute temperature.

8. Describe some practical form of constant volume air-thermometer: state clearly what measurements you would make to obtain the coefficient of increase of pressure with rise of temperature: and what precautions should be taken that an accurate result might be obtained.

What is the absolute zero of the air-thermometer?

9. A quantity of gas is collected in a graduated tube over mercury. The volume of the gas at $10^{\circ}\text{C}.$ is 50 c.c. and the level of the mercury in the tube is 10 cm. above the level outside; the barometer stands at 75 cm. Find the volume which the gas would occupy at $0^{\circ}\text{C}.$ and 76 cm. barometric pressure.

10. A quantity of air 3 c.c. in volume at atmospheric pressure is introduced into the space above a barometric column which originally stands at 760 mm. The column is depressed by 190 mm.; find the volume occupied by the air.

11. A quantity of dry air occupies 1000 c.c. at 20°C . and under a pressure of 760 mm. of mercury. At what temperature will it occupy 1400 c.c. under a pressure of 750 mm. of mercury?

12. The density of air at 0°C . and 760 mm. pressure is 1.29 grammes per litre. What is its density at 491°F . and 1000 mm. pressure?

13. A mass of gas occupies a volume of 35 c.c. at a pressure due to 75 cm. of mercury and temperature of 15°C ., find its volume at a pressure due to 76 cm. of mercury and temperature 0°C .

14. When the height of the barometer is 76 cm. the volume of a given mass of gas is 100 c.c., the temperature being 0°C . When the temperature is raised to 100° , and the pressure increased by that due to 28 cm. of mercury, it is found that the volume is the same; find the coefficient of increase of pressure of the gas.

15. The pressure of a mass of air kept at constant volume increases by that due to 2.78 mm. of mercury for a rise of temperature of 1°C . If the pressure at 0° be that due to 760 mm., find the temperature when the pressure is that due to 899 mm.

CHAPTER VIII.

CHANGE OF STATE. SOLID TO LIQUID.

106. Fusion of a Solid. Melting-point. If heat be applied to a solid body, its temperature rises, and the body usually expands, until a certain temperature is reached at which the body begins to melt. This temperature is called the fusing-point; the further application of heat produces no rise of temperature, until the body is melted, and the whole has become liquid.

The latent heat of fusion of a solid has already been defined as the quantity of heat required to change 1 gramme of the solid to a liquid without change of temperature, and the method of determining the latent heat of fusion of ice has been explained.

EXPERIMENT (25). *To find the melting-point of paraffin wax.*

Take a piece of glass tubing, draw out one end to a fine point and close it. Place some small fragments of paraffin wax in the tube; melt it by placing it in hot water and then let it solidify so as to fill the tube. Fasten the tube to a thermometer, with an india-rubber band or otherwise, so that the narrow part of the tube may be alongside the bulb. The wax when solid is opaque. Place the tube in a beaker of water, and gradually warm the water, keeping it stirred, noting the temperature. When the temperature of the melting-point is reached, and the wax becomes fluid, it loses its opacity, becoming transparent; observe carefully the reading of the thermometer when this occurs; allow the

bath to cool, and note the temperature at which the wax again becomes opaque. If the heating has been sufficiently slow, the two temperatures will differ very slightly; the mean of the two may be taken as the melting-point.

A similar method may be employed to find the melting-point of many other substances.

Many bodies, however, do not change suddenly from the hard solid to the liquid state and vice versa. Thus glass can exist in a soft plastic state over a wide range of temperature and in consequence of this can be worked and moulded.

Iron has a plastic condition, just below the temperature at which it melts, and when in this state can be welded. Plumber's solder remains a viscous substance while cooling through a considerable number of degrees.

The plasticity of ice is much greater just below the melting-point than it is at a lower temperature.

Such bodies become soft or plastic solids before melting; there is however a temperature at which a sudden absorption of heat occurs and the plastic solid becomes a fluid. This temperature is the melting-point.

107. Change of Volume on melting. Some bodies contract on melting, others expand. Ice, iron, lead, brass, all occupy a greater volume in the solid than in the liquid state. It is in consequence of this that sharp castings may be taken of these substances; thus ice floats on water, and solid lead in the liquid metal.

Other bodies, such as wax and bismuth, expand when melted. The contraction of paraffin wax is easily shewn by filling a test tube with melting wax and allowing it to solidify; the solid wax does not nearly fill the tube.

EXPERIMENT (26). *To determine the contraction of ice on melting.*

Take a test tube holding about 20 c.cm. Fit it with a good cork, through which a tube passes; the bore of the tube should be about 5 mm. and its length some 12 or 15 cm. Measure the bore of the tube, and find the area of its cross-

section. Determine also the volume of the test tube up to the cork. This may be done by letting water run in from a burette. Fill the test tube with cold water which has been well boiled. Insert the cork and attach a millimetre scale to the tube. Freeze the water in the test tube by putting it in a freezing mixture. This must be done gradually and carefully, so as to freeze the water from the bottom, otherwise the expansion on freezing will burst the test tube. As the freezing proceeds the water which at first was just visible in the glass tube above the cork rises up the tube; note the distance it has risen when the freezing is complete; and, by multiplying this by the area of the tube in square centimetres, find the increase in volume of the contents of the tube on freezing. When all the water in the test tube has become solid repeat the observation in the reverse order by melting the ice and observing the fall of the water.

108. Relation between Melting-point and Pressure. The melting-point of ice and of other bodies which contract on melting is lowered by increasing the pressure to which they are exposed.

Thus at a pressure of about 160 kilogrammes per square cm., or nearly 160 atmospheres, ice will melt at -1°C . instead of at 0°C .

This lowering of the freezing-point may be shewn by the following experiments.

(1) An iron cylinder *AB*, fig. 46, is closed with a strong screw *C*. It is then filled with water and frozen; a metal ball is placed on the top of the water and the cylinder closed by the screw. The whole is then covered with ice, the ball being at the top of the cylinder; pressure is then applied by means of the screw. On opening the cylinder, the water inside is found to be still frozen, but the ball is at the bottom. The water has become liquid under the pressure and the ball has sunk to the bottom; on removing the pressure the water has again frozen.

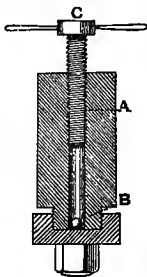


Fig. 46.

(2) Take a strong wooden mould of any form, such as a hollow cylinder with a wooden piston or plunger, pack it tightly with snow or powdered ice; press the plunger firmly down by means of a small hydraulic press or heavy weights. On again opening the cylinder after removing the pressure it will be found to contain a solid cylinder of clear ice.

109. Regelation. This same process is illustrated every time a snowball is made. When the snow is too cold it will not bind. When it is near its melting-point, comparatively little pressure is needed to make it melt; on removing the pressure the water formed freezes again and the snow binds. This phenomenon is known as regelation.

EXPERIMENT (27). *To observe the effects of regelation.*

(1) Place two pieces of ice in water and press them strongly together; remove the pressure; the two pieces are frozen together.

(2) Support a block of ice on the ring of a retort stand or in any convenient manner. Fasten one end of a stout copper wire to the table; pass the wire over the ice and attach a heavy weight, 10 or 20 kilos, to the other end. It will be found that the wire cuts its way through the ice, but that water formed freezes up again above the wire so that the continuity of the ice remains unbroken, though the wire passes through.

Underneath the wire the ice is subject to considerable pressure; it therefore melts, although at the temperature of the freezing-point or even a little below it. The water formed flows round the wire to the upper surface where it is free from pressure. But heat is required to melt the ice and the copper wire is a good conductor¹, hence heat is conducted from the water above the wire through the wire to melt the ice. The water is at the temperature of the freezing-point and therefore the abstraction of heat from it causes it again to freeze.

Thus the heat required to melt the ice below the wire is obtained from the freezing of the water above the wire.

¹ See Chapter x., § 145.

In substances such as beeswax or paraffin which expand on melting the opposite effects are produced; the melting-point is raised by pressure.

110. Freezing Mixtures. Heat is required to melt a solid and the action of many freezing mixtures depends on this fact.

EXPERIMENT (28). *To observe the fall of temperature produced by melting.*

(1) Drop a few crystals of ammonium nitrate into a beaker of water in which a thermometer is placed; the temperature will be observed to fall as the crystals melt. The action of the water melts the crystals. Energy is needed to produce the change from solid to liquid and the energy is obtained from some of the heat present in the water, which is thereby lowered in temperature.

(2) Break some pieces of ice up into small fragments and mix them with salt; the ice melts, absorbing the latent heat of fusion from the mixture, and the temperature is reduced to between -15°C. and -20°C. Fahrenheit's zero, 32°Fah. below freezing-point, was obtained in this manner.

111. Development of Heat by (1) Chemical Change. Heat is developed by many chemical combinations; it may happen that when a body is dissolved in water the product combines with the water and develops heat; if this heat exceeds the heat required for the solution a rise of temperature will be the result. This is the case when caustic potash is dissolved in water.

(2) *Solidification.* Again, heat is developed in the act of solidification. Water was cooled by Despretz in fine tubes some 20° below freezing-point. When this cooling has been produced, solidification may be caused by a slight disturbance. Ice is formed and the temperature rises to the freezing-point.

EXPERIMENT (29). *To observe the development of heat produced by solidification.*

Make a very strong solution of acetate of soda by heating it with water in a flask; close the flask with a plug of cotton

wool and allow the solution to cool; it can be cooled with care to temperatures very considerably below that at which it would normally become solid. Remove the cotton wool and insert a thermometer, this will very often be sufficient to produce solidification, if not drop in a small crystal. The mass solidifies at once and the temperature rises very considerably.

112. Laws of Fusion. We may thus sum up our results with the following laws of fusion.

(1) *A substance begins to melt at a temperature, which is constant for the same substance, if the pressure be constant, and is called the melting-point.*

(2) *The temperature of the solid remains unchanged while fusion is taking place.*

(3) *If a substance expand on solidifying, like ice, its melting-point is lowered by pressure; if it contract, like wax, its melting-point is raised by pressure.*

(4) *The latent heat of fusion of a substance is the quantity of heat absorbed by unit mass in changing from the solid to the liquid state without change of temperature. This amount of heat is constant for a given body melting at a given temperature.*

It should also be remembered that the effect of pressure in changing the melting-point of a solid body is very small, being in the case of ice about $\frac{1}{180}$ of degree Centigrade for one atmosphere.

EXAMPLES.

FUSION AND SOLIDIFICATION.

1. In what way does pressure alter melting-points? Consider the cases of ice, cast iron, wax, and phosphorus.

2. Explain the way in which two blocks of ice at 0°C . may be made to unite by the application of pressure.

3. Give a sketch of the formation and motion of glaciers. Describe any experiments with ice, which illustrate the subject. Account for the fact that a snowball by sufficient pressure may be converted into a ball of clear ice.

4. Describe an experiment to prove that ice contracts on melting. A copper wire is passed round a block of ice and supports a heavy weight. It is found that the wire eats its way through the ice but that the latter freezes up again behind the wire. Explain this.

5. A pound of ice at -10°C . is heated under atmospheric pressure to 200°C . Trace the changes of volume and state which it undergoes, and shew how to calculate the amount of heat required to produce them, giving numerical values where you can.

6. Describe in detail the process of freezing of a pond until the ice is strong enough to support heavy loads. What would be the effect on its bearing power of breaking the ice all round the edge?

7. Explain the adhesion of two pieces of melting ice when pressed together and let go. Why is it not readily possible to make snowballs during hard frost?

8. Describe the changes that a pound of ice undergoes in being heated from -10°C . to 60°C .: and shew what amount of work would have to be done to supply the heat necessary to raise its temperature through this range.

[The specific heat of ice is $\cdot 5$, the latent heat of fusion of ice 79.

The mechanical equivalent of heat 1390 foot-pounds.]

9. It is said that the heat received on the earth from the sun would, if uniformly distributed, melt in 2 hrs. 13 minutes a shell of ice over the earth 2.5 cm. in thickness. Supposing all the heat which falls on it to remain in the water thus formed, how long would it take to turn it into steam; the latent heat of fusion of ice being 79, and that of evaporation of water 537? The radius of the earth may be taken as 6440 kilometres and the specific gravity of ice as $\cdot 9$.

CHAPTER IX.

CHANGE OF STATE. LIQUID TO GAS.

113. Evaporation. If a small quantity of water be exposed in a flat dish or saucer, in a fairly dry room at ordinary temperatures, the water disappears in a short time and the dish becomes dry. If the experiment be repeated with ether or alcohol the result is the same but the dish dries much more rapidly. The liquid has passed gradually into the gaseous state, and this process when it goes on slowly from the surface of the liquid is called evaporation.

The term is also occasionally applied to solids. Thus snow sometimes disappears in dry frosty weather, the water-substance passes from the solid to the gaseous state without becoming liquid between, but this is more properly called sublimation.

DEFINITION. *Evaporation is a gradual change of a substance from the liquid to the gaseous state which takes place at the surface of the substance at all temperatures.*

A substance which passes readily into the gaseous state is called volatile.

The substance when in the gaseous state, produced thus by evaporation, is called a vapour. The distinction between a vapour and a gas will be considered in what follows and a definition of a vapour will be given in § 119.

114. Pressure of Vapours. When a volatile liquid such as water or ether is placed in a closed vessel containing

dry air, evaporation takes place and the quantity of vapour in the vessel is increased. The vapour formed exerts pressure like a gas and hence the pressure on the sides of the vessel is increased. The formation of vapour continues until the pressure which it exerts has reached a certain limiting value depending on the nature of the substance which is evaporating and on the temperature. This limiting value does not depend on the size or shape of the vessel which contains the vapour, or upon the nature of the gas, air, hydrogen, or whatever it may be which the vessel contains in addition to the vapour, provided (1) that there is no chemical action between this gas and the vapour, and (2) that some liquid remains in the vessel.

When the pressure exerted by the vapour which a space contains at a given temperature has reached the limiting value for that temperature, the space is said to be *saturated* with the vapour.

When a space is saturated with vapour the pressure exerted by the vapour is known as the saturation pressure of the vapour.

If the volume of a space which is saturated with vapour be reduced, the pressure of the vapour is not altered, some of the vapour is condensed to the liquid state but the pressure remains the same; if the volume of the space be increased, provided that it contains sufficient liquid, more liquid evaporates and the pressure exerted by the vapour soon¹ attains the same value as before the alteration of volume.

If the temperature of a space containing liquid and saturated with its vapour be raised², more liquid is evaporated and the saturation pressure of its vapour is increased; if on

¹ Since however evaporation takes time the pressure immediately after the increase in volume will be too small, but it will quickly rise to its former value.

² If the space also contains air or some other gas it must be remembered that when the temperature is changed the pressure of this gas is altered. The total observed change of pressure is due to the change in the pressure of the gas together with the change in the pressure of the vapour, and in finding the latter change the change in the pressure of the gas must be allowed for.

the other hand the temperature falls, some of the vapour is condensed to liquid and the saturation pressure is less than previously.

***115. Experiments on Vapour Pressure.** In making experiments on the pressure of vapours the apparatus shewn in fig. 47 will be found useful.

It is a modification of that used in § 95 for verifying Boyle's Law.

The tube AB shewn in fig. 41 is replaced by a tube AB , fig. 47, having two stopcocks S_1 S_2 at the upper end. These are placed one above the other with a short length of glass tubing between them. The volume of the tube between S_1 and S_2 may conveniently be about .25 c.cm. Above the upper stopcock is a funnel. When the upper stopcock is open the funnel and space between S_1 and S_2 may be filled with water or any other liquid which is to be examined. Close the upper stopcock S_1 and open S_2 ; the liquid in S_1S_2 runs down into the space AB , and on evaporation exerts pressure on the mercury in AB . This pressure can be measured by the aid of the sliding piece CD , fig. 41. The tube AB can be surrounded by a wider glass tube forming a jacket; by putting warm water into this jacket the temperature of the vapour can be varied and the laws of the variation of its pressure with temperature measured.

With this apparatus the following experiments may be performed.

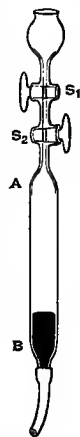


Fig. 47.

EXPERIMENT (30). *To measure the pressure of water vapour at a given temperature.*

Adjust the apparatus, which should be absolutely dry, so that, when the taps S_1 S_2 are open, the mercury may stand at the same level in AB and CD of figure 41. The pressure of the air in AB is then the atmospheric pressure. Close S_2 , put some water in the funnel above S_1 and so fill the space S_1S_2 with water¹.

¹ The air sticks in the tube between the stopcocks and care is necessary to fill the tube.

Note the level of the mercury in the tube AB . Close S_1 and open S_2 .

The water runs down into AB and some of it evaporates; the pressure in AB is increased and the mercury is driven down in AB and rises in CD .

Wait for some little time, to allow the space to become saturated, and make sure that some water is left uncondensed in AB ; if no water is visible, introduce some more in the manner just described. The difference in level between the mercury columns in AB and CD does not measure the pressure of the vapour directly, because the air in AB now occupies a greater volume than previously; its pressure is therefore less than it was. Raise CD until the mercury in AB comes back to its original level. The air in AB has now the same volume and pressure as before, the height of the mercury in CD above that in AB measures the pressure due to the vapour.

Thus the saturation pressure of aqueous vapour at the temperature of the experiment and in presence of air at atmospheric pressure is measured.

EXPERIMENT (31). *To observe the effect of change of temperature on the saturation pressure of a vapour.*

Place some warm water at a temperature say of 30°C . in the jacket tube.

More of the water in AB evaporates, the pressure of the vapour is increased, and the column of mercury in CD is forced up. Adjust the height of CD until the level in AB is the same as before, and measure the height of the column in CD above AB . This gives the excess of the pressure in AB over the atmospheric pressure.

Now this excess is due to (1) the increase of the pressure of the air in AB produced by a rise of temperature at constant volume, and (2) the vapour pressure of the liquid. The increase due to the change of temperature can be calculated as described in § 102. It will be $p_0 \{T/T_0 - 1\}$, where p_0 is the atmospheric pressure, T_0 the absolute temperature at which the stopcocks were closed, T the absolute temperature

at the time of the observation¹. Subtract this from the total observed increase: the difference gives the vapour pressure of the liquid at the temperature of the observation.

On making the observations it will be found that the vapour pressure increases greatly as the temperature rises.

EXPERIMENT (32). *To shew that the vapour pressure is independent of the pressure of the air present.*

Let the space AB be filled with dry air, the mercury in the tube being also dry. Open the stopcocks and adjust the level of CD , so that the mercury is at the same height in the two tubes. Close the stopcocks and raise CD until the air is compressed to about half its volume. Note the level of the mercury in AB in this case and also the level of that in CD . Fill the funnel and tube with liquid, and proceed with the experiment as before, adjusting CD so that the level in AB remains the same. The increase of the height of the mercury column in CD over its height before the liquid was admitted to AB gives the vapour pressure in the presence of air at twice the atmospheric pressure.

It will be found that the pressure is the same as that observed previously at the same temperature.

By starting in the same way with the level in CD below that in AB we can experiment when the air is at less than atmospheric pressure.

If the slide be sufficiently long we can with the same apparatus examine the effect of vapour pressure in a vacuum. For this purpose it must be possible to lower CD some 80 or 85 cm. below the top of AB .

Open the taps S_1 and S_2 and raise CD until the mercury rises above S_2 ; close S_2 and lower CD .

If CD be lowered sufficiently so that the difference in level between S_2 and the top of the column in CD may be greater

¹ By altering slightly the method of observation, and filling AB first with dry air at atmospheric pressure and at the higher temperature 30° and then admitting the moisture, the necessity for this correction may be avoided.

than the barometric height, the mercury will fall slightly in *AB*, leaving a vacuum between the top of the column and the stopcock.

Boil some water for some time so as to free it from air; allow it to cool, and introduce it in the manner already described above the mercury in *AB*. The mercury is depressed; adjust *CD* so as to bring the column in *AB* up to the same mark as previously, and then measure the pressure of the vapour at the temperature of the observation. It will be found to be the same as that observed previously at the same temperature when there was some air in *AB*¹.

EXPERIMENT (33). *To shew that the vapour pressure at a given temperature due to two vapours which do not act chemically on one another is the sum of the vapour pressures due to each liquid separately at the same temperature.*

Observe in the manner described in Experiment 30 the vapour pressures due to each of the two liquids, say benzene and water. Mix the two liquids, introduce the mixture into the space above the mercury, and observe the pressure again. It will be found to be the sum of the pressures previously observed. The temperature must be kept the same throughout the experiment.

116. Pressure of water vapour at various temperatures. For experiments on vapour pressure in a vacuum the following arrangement of apparatus is more convenient. A tube about 80 cm. long and 1 cm. in diameter *AB*, fig. 48, is cleaned and dried, closed at one end, and then filled with clean dry mercury, and inverted with the open end in a cistern of mercury. A barometer is thus formed and the mercury in the tube sinks to the barometric height, about 76 cm., above that in the cistern.

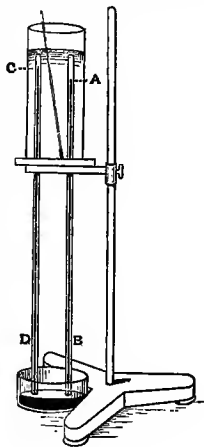


Fig. 48.

¹ For observations on vapour pressure in a vacuum, the form of apparatus described in § 116 is preferable.

A small quantity of the liquid is placed in a pipette, the lower end of which is bent upwards, so as to be inserted in the mercury in the cistern with its open end below the barometer tube; by blowing gently into the pipette a small quantity of the liquid can be caused to rise in drops through the column to the surface of the mercury. This liquid evaporates into the vacuum above the column which is depressed, and the depression measures the vapour pressure. By surrounding the tube with a jacket which can hold water and raising the temperature of this water, the vapour pressure at various temperatures given by the thermometer in the jacket can be found.

In order to allow for changes in the height of the barometer during the experiment it is usual to set up two barometer tubes side by side in the same water-jacket, and to introduce the liquid into one only.

The difference in height of the two columns gives the vapour pressure directly. This second tube is shewn at *CD* in the figure. To measure this difference in height a vertical scale may be set up alongside the tube, or for more accurate work a kathetometer may be used. This was the method employed by Regnault in his researches on the vapour pressure of water at low temperatures.

The scale is necessarily at some little distance from the tubes and it is not easy therefore to read the height exactly. To get over this difficulty it is convenient to fix a piece of card one edge of which can be set horizontally a little way in front of the tubes. The observer then places his eye so that the top of the mercury column is just hidden by the card, and then keeping his eye fixed reads the graduation of the scale which coincides with the card; in applying this method it is necessary that the tube and the scale should be at the same distance from the card. Greater accuracy will be secured if a reading telescope is used to view the scale and column.

EXPERIMENT (34). *To compare the vapour pressure of various liquids at a given temperature.*

Fit up side by side in the same jacket a number of barometer tubes as described above; allow one to remain as a barometer.

Introduce into each of the others respectively small quantities of the liquids to be experimented on, taking care to

introduce in each case sufficient to allow some of the liquid to remain unevaporated on the top of the mercury columns.

Measure the differences in height between the barometer column and the columns in the other tubes.

These differences give in each case the pressure, in mm. of mercury, of the vapours in the respective tubes.

EXPERIMENT (35). *To measure the pressure of aqueous vapour at different temperatures.*

Fit up as in the last experiment two barometer tubes in a

PRESSURE OF AQUEOUS VAPOUR IN MM. OF MERCURY.

$t^{\circ}\text{C.}$	mm.	$t^{\circ}\text{C.}$	mm.	$t^{\circ}\text{C.}$	mm.	$t^{\circ}\text{C.}$	Atmos.
-10	2.08	16	13.54	90	525.39	100	1.0
-9	2.26	17	14.42	95	633.69	110	1.4
-8	2.46	18	15.36	99	733.21	120	1.96
-7	2.67	19	16.35	99.1	735.85	130	2.67
-6	2.89	20	17.39	99.2	738.50	140	3.57
-5	3.13	21	18.50	99.3	741.16	150	4.7
-4	3.39	22	19.66	99.4	743.83	160	6.1
-3	3.66	23	20.89	99.5	746.50	170	7.8
-2	3.96	24	22.18	99.6	749.18	180	9.9
-1	4.27	25	23.55	99.7	751.87	190	12.4
0	4.60	26	24.99	99.8	754.57	200	15.4
1	4.94	27	26.51	99.9	757.28	210	18.8
2	5.30	28	28.10	100	760.00	220	22.9
3	5.69	29	29.78	100.1	762.73	230	27.5
4	6.10	30	31.55	100.2	765.46		
5	6.53	35	41.83	100.3	768.20		
6	7.00	40	54.91	100.4	771.95		
7	7.49	45	71.39	100.5	773.71		
8	8.02	50	91.98	100.6	776.48		
9	8.57	55	117.48	100.7	779.26		
10	9.17	60	148.79	100.8	782.04		
11	9.79	65	186.94	100.9	784.83		
12	10.46	70	233.08	101	787.59		
13	11.16	75	288.50	105	906.41		
14	11.91	80	354.62	110	1075.37		
15	12.70	85	433.00				

water-jacket. Introduce some water above the mercury in one. Place a thermometer and stirrer in the bath. Vary the temperature of the water and take a series of readings of the difference of level of the two columns, and of the corresponding temperatures, keeping the bath well stirred. Enter the results, which give in mm. of mercury the pressure of the aqueous vapour at the different temperatures, in a Table, as shewn on the previous page.

This Table gives the results of Regnault's experiments on the pressure of aqueous vapour at varying temperatures from -10°C. to 230°C.

117. Dalton's Laws for vapours. By means of experiments such as those described the laws regulating the pressure of vapours known as Dalton's Laws have been established. They may be stated thus.

(1) *The saturation pressure of a vapour depends only on its temperature.*

(2) *The pressure of a mixture of gases and vapours, which have no chemical action on each other, is the sum of the pressures, which each would separately exert, if it were alone in the space occupied by the mixture.*

Example. *The pressure of a mass of air saturated with aqueous vapour at 15° is observed to be 756 mm.; find the pressure due to the air alone.*

The saturation pressure of aqueous vapour at 15° is 12.7 mm. Hence the pressure due to the dry air is $756 - 12.7$ or 743.3 mm.

***118. Unsaturated vapours.** The first of the above laws applies only to saturated vapours, the pressure exerted is the maximum pressure which can be exerted at the temperature; some of the liquid must be present, so that if the temperature rise more vapour can be formed.

If the pressure of a vapour at a given temperature is not the maximum which it could exert at that temperature, the vapour is said to be unsaturated.

Thus, if a *small* quantity of water be placed in a *large* space filled with dry air, the whole of the water will evaporate

without the pressure reaching the saturation pressure. The vapour is unsaturated. Under these circumstances the vapour obeys approximately the gaseous laws; if the volume of the space increase the vapour pressure decreases approximately according to Boyle's Law; when the volume is doubled the vapour pressure is halved, and so on. If the volume decrease the vapour pressure increases in the inverse ratio, until the maximum or saturation pressure for the temperature is reached. When the pressure has risen to its maximum value no increase of pressure is produced by a further diminution of volume; some of the vapour is condensed, the pressure remaining unchanged.

Similarly, if the temperature be raised, the volume remaining unaltered, the pressure increases in accordance with Charles' Law, being proportional to the absolute temperature. If the temperature be reduced, the pressure falls in accordance with the same law, until, at a certain temperature, it reaches the saturation pressure for that temperature; for a further reduction of temperature the pressure falls more quickly than it would do were it still to follow Charles' Law; some of the vapour is condensed to liquid, and, as the temperature is further reduced, the pressure falls, remaining always at the saturation pressure for the given temperature.

These results may be indicated on a diagram thus:

Take two lines OA , OB , fig. 49, at right angles and let the volume be measured by lines parallel to OA , the pressure by lines parallel to OB .

Let OL represent the saturation pressure at the temperature of the experiment, and draw LK parallel to OA .

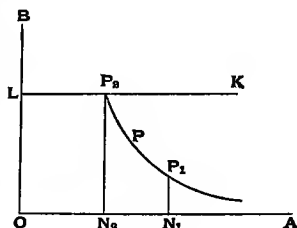


Fig. 49.

Let ON_1 represent the volume of a quantity of unsaturated

vapour, and N_1P_1 its pressure. As the volume of the vapour is reduced, the pressure increases and the relation between the two is shewn by the curve line P_1PP_2 , which is approximately the same as that for a gas.

This continues until a volume, ON_2 in the figure, is reached at which the curve line cuts LK . The pressure becomes equal to the saturation pressure at that temperature. As the volume is still further reduced, there is no increase in pressure until the whole is condensed to liquid; the relation between pressure and volume is shewn by part of the horizontal line KL . The line P_1P_2L is called an isothermal line.

Again, in fig. 50, let the horizontal line OA represent temperatures, and the vertical line OB pressures.

Corresponding to each temperature measure off vertical lines to represent the saturation pressure; the ends of these lines will lie on a curve such as LP_2K .

Let N_1P_1 be the pressure of an unsaturated vapour corresponding to a temperature ON_1 . The relation between pressure and temperature will be given by a straight line P_1P_2 which would cut AO produced at a point corresponding to the temperature of -273°C .

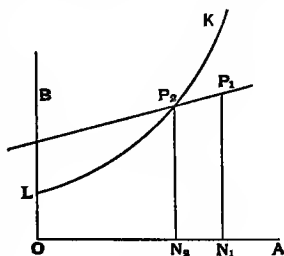


Fig. 50.

Let this line cut LK in P_2 , and let ON_2 be the temperature corresponding to P_2 . Then, until the temperature falls to ON_2 , the relation between pressure and temperature is given by the straight line P_1P_2 ; if the temperature be still further reduced, the relation will be given by the curve P_2L .

Examples. (1) The pressure of a mass of aqueous vapour 100 c.cm. in volume is 4.35 mm., the temperature being 20° ; find the pressure when the volume is 50 c.cm., and also the volume at which the gas becomes saturated, the maximum pressure being 17.4 cm.

If the vapour obey Boyle's Law, the pressure when the volume is 50 c.cm. will be double that when the volume is 100 c.cm., or 8.7 mm. This is less than the saturation pressure and therefore is the pressure required. Again, assuming Boyle's Law to hold up to the saturating pressure, the volume when the pressure is 17.4 is

$$100 \times 4.35 / 17.4 \text{ or } 25 \text{ c.cm.}$$

Thus, when the volume has been reduced to 25 c.cm. the vapour becomes saturated.

(2) A given space is just saturated with vapour at 15°C ., there being no liquid present; find the pressure exerted by the vapour at 30° .

At 15° C. the pressure is that due to 12.7 mm.; from 15° to 30° the vapour obeys Charles' Law approximately. Thus

$$\begin{aligned}\text{Pressure at } 30^{\circ} &= \frac{12.7 \times (273 + 30)}{273 + 15} \\ &= 13.36 \text{ mm.}\end{aligned}$$

*119. Vapours and Gases. Critical temperature.

So far, the distinction we have drawn between vapours and gases has been that a vapour can be liquefied by reducing its volume, while, for the temperatures at which we have supposed the experiments conducted, a gas continues to obey Boyle's Law for all pressures; this distinction for most practical purposes is sufficient.

But by suitable devices a gas can be liquefied. Thus Faraday liquefied nearly all the gases known to him except oxygen, hydrogen, marsh-gas, and carbonic oxide. One simple arrangement of apparatus used by him is shewn in fig. 51. It consists of a strong bent tube. Some substance from which the gas to be experimented on can be evolved by heating is placed at one end, and the tube is hermetically sealed. The other end is placed in a freezing mixture of ice and salt. On heating the tube the gas is given off, the pressure becomes very great and the gas is condensed to liquid in the cold end.

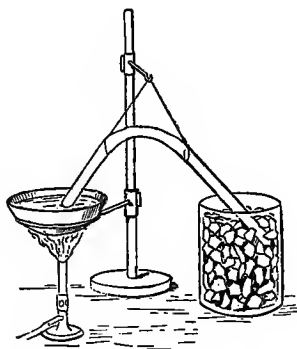


Fig. 51.

Dr Andrews liquefied carbonic acid gas by filling a strong glass tube with it, and allowing the open end of the tube to communicate with a reservoir of mercury. By means of a screw plunger the mercury was forced up the tube, thus compressing the gas. Carbonic acid gas, at a temperature of about 13° C., can be liquefied at a pressure of about 48 atmospheres.

By the aid of this apparatus Andrews was able to shew

that, so long as the temperature of this substance was below $30^{\circ}9\text{ C.}$, it could be liquefied by pressure, but if its temperature exceeded this value no pressure however great reduced it to the liquid state. This temperature is called the critical temperature for carbonic acid. Below its critical temperature carbonic acid gas has the properties of a vapour; it can be liquefied by the application of sufficient pressure; above this temperature pressure will not liquefy it. When near its point of condensation, carbonic acid does not obey Boyle's Law at all accurately; at some distance from this point its pressure at a given temperature is, in accordance with the law, very nearly proportional to its density.

Other gaseous substances have their critical temperatures; that of ether is about 187° C. while for water it is over 400° C. These substances in the gaseous form then at ordinary temperatures are vapours. They can be condensed by pressure to liquids, for at ordinary temperatures they are below their critical temperature.

On the other hand, the critical temperatures of the so-called permanent gases are extremely low, that for oxygen may be about -130° C. , for nitrogen -167° C. , but the values are difficult to determine. These substances at ordinary temperatures are above their critical temperature; they cannot be liquefied by pressure only; to produce liquefaction they must be cooled down to below the critical temperature, and since their vapour pressure is enormously high they require even at the low temperature great pressure to produce liquefaction. The temperature corresponding to the point P_2 in fig. 50 is very low, while the pressure is high.

Definition of critical temperature. *There is for all substances in the gaseous state a temperature such that the substance can be liquefied by pressure only if it be below this temperature, and cannot be liquefied if it be above this temperature. This temperature is called the critical temperature.*

A substance in the gaseous state below its critical temperature is called a vapour, above this temperature it is a gas.

Commonly however the term vapour is used, in a more restricted sense, for a substance in the gaseous state, which, at

ordinary temperatures, can be liquefied by moderate pressure. Thus ether vapour is spoken of as a vapour, for it can be liquefied by a pressure of about half an atmosphere at temperatures of from 12° to 15° C.

Carbonic acid gas can be liquefied also at these same temperatures by pressure only, but the pressure required is some 50 atmospheres, about 100 times that needed to liquefy ether. Carbonic acid gas is therefore commonly, though not strictly, spoken of as a gas.

120. Ebullition or Boiling. A reference to the Table on p. 130 shews that at the temperature of 100° C. the pressure of aqueous vapour is 760 mm. of mercury. Now the standard atmospheric pressure is 760 mm., and water is seen to *boil* at this pressure and temperature. Thus in this case water boils at the temperature at which the pressure of its vapour is equal to the pressure to which the liquid is subject.

We proceed after some necessary definitions and explanations to describe some experiments to shew that this rule holds generally.

Definition of Boiling or Ebullition. *Boiling or ebullition is the change of a substance from the liquid to the gaseous state, which takes place throughout the mass of the liquid.*

Thus vapour is formed both by evaporation and by boiling; in the first case however the change takes place at the surface of the liquid only, in the second case it proceeds throughout the mass of the liquid.

When heat is applied to a mass of liquid, such as a quantity of water in a beaker placed over a Bunsen burner, the lower layers of liquid are first warmed. These expand and rise to the surface, their place is taken by the colder layers from above, and by this process the mass is warmed through; the air which is contained in the water expands, as the temperature is increased, and rises in bubbles to the surface. After a time the temperature of the lower layers is raised up to or slightly above 100° C. Water vapour or steam is formed as bubbles which rise into the colder layers above and are condensed, causing the water to "simmer," whilst the steam

parts with its latent heat of evaporation, warming the water, and mixing up the layers until the whole mass reaches 100°C .

When this stage has been reached the steam rises to the surface and escapes into the air, and the simmering noise ceases. The water is boiling; the steam is formed throughout the mass, because water vapour at 100° exerts a pressure just equal to that of the standard atmosphere.

EXPERIMENT (36). *To shew that the pressure of water vapour when boiling is equal to the atmospheric pressure.*

(a) Fill a barometer tube with mercury as described in § 116. Introduce some water above the mercury, and jacket the whole tube as shewn in fig. 52 with a wide glass tube. Connect the upper end of this steam jacket to a boiler and allow the steam to escape from the jacket through a pipe at its lower end. Admit the steam gradually into the jacket: as the temperature rises, the pressure of the water vapour in the tube increases, and the mercury column is depressed, until when the whole has become steady the level of the mercury is the same¹ inside the tube and in the cistern. The pressure exerted by the water vapour is equal to the atmospheric pressure.

(b) The following experiment illustrates the same point.

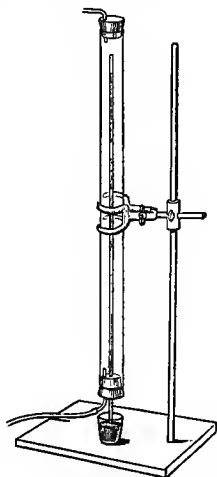


Fig. 52.

ABC, fig. 53 and 53 *a*, is a bent tube closed at *C*, open at *A*, the arm *AB* being the longer. The arm *BC* contains some water freed from air by boiling; below the water is mercury filling the lower part of the tube and rising in the arm *AB*.

¹ If there is a large quantity of water left in the tube, owing to the weight of this water, the mercury inside the tube will be slightly below that outside.

The level of the mercury in this arm is below that in *BC*. The water and mercury completely fill *BC*. The tube *AB* passes through a cork closing a flask. The flask contains water which can be boiled, and the steam as it rises surrounds the lower part of the tube and escapes through a hole in the cork. The upper part of the neck of the flask can if necessary be jacketed as shewn in fig. 6.

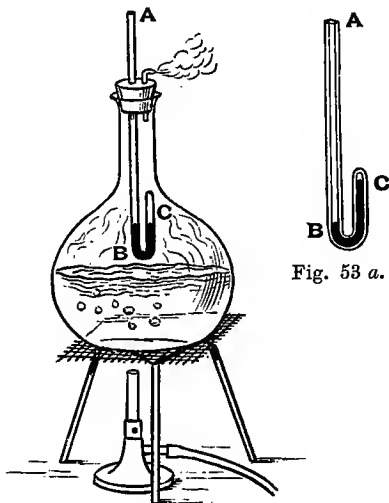


Fig. 53.

After the water has been boiling for some time the temperature of the water in the tube *BC* rises to 100° C. and the pressure of the vapour formed there forces the mercury down *CB* and up *BA* until it stands at the same level in the two arms. Thus the pressure of the vapour in *BC* is equal to the pressure on the mercury in *AB*, that is to the atmospheric pressure.

Thus we see that water boils at a temperature at which the pressure of its vapour is equal to the atmospheric pressure.

121. Boiling under diminished pressure. Assuming the law which has just been enunciated to hold generally we should expect that by reducing the pressure the temperature at which water boils may be reduced also; observation and experiment both shew this to be the case.

Thus on the top of a mountain the temperature of boiling water is much less than 100°C . On Mont Blanc the temperature of the boiling-point is about 85°C . The following experiments illustrate this point.

EXPERIMENT (37). *To shew that the boiling-point of water is reduced by reducing the pressure.*

(1) Boil some water in a beaker or flask, place it under the receiver of an air-pump, wait till the water has ceased to boil and then exhaust rapidly; the boiling recommences.

(2) Boil some water in a strong Florence flask; when the water is boiling freely remove the burner and cork up the flask tightly. Invert it so that the water may cover the cork, as in fig. 54, and wait till the boiling ceases. The space above the water is filled with water vapour at a fairly high temperature, and the water is subject to the pressure of this vapour. Allow some drops of cold water to fall from a sponge on to the flask which is thereby cooled; some of the vapour is condensed, the pressure on the water surface is reduced and the boiling recommences.

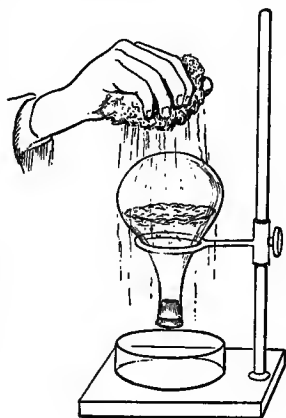


Fig. 54.

EXPERIMENT (38). *To measure the temperature of steam issuing from water boiling under various pressures.*

Take a retort flask *AB* in fig. 55. At *B* there is an aperture through which a thermometer passing through a tightly fitting cork can be inserted. *A* is connected by means

of a glass tube *C* and india-rubber tubing to an air-pump. A water suction pump which will work continuously is best. At *C* a side tube sealed into the glass tube, thus forming a T-piece, passes through a cork into a small flask or beaker. This cork fits tightly and a second tube, bent twice at right angles, passes

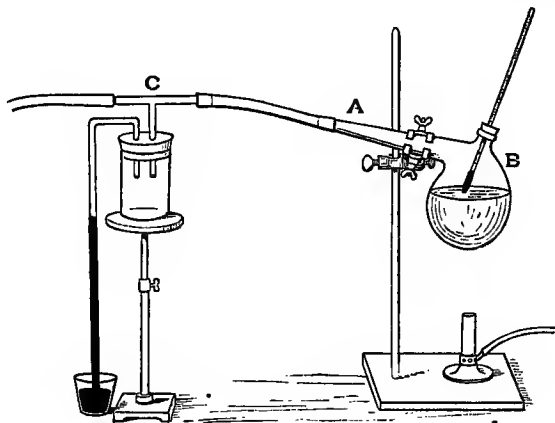


Fig. 55.

through it and has its open end immersed in a vessel of mercury, thus forming a pressure gauge. This tube must be longer than the height of the barometer and should have a scale attached.

Heat the water in the flask *AB* by a Bunsen burner and start the pump. The pressure of the air above the water is thus reduced, and the mercury rises in the gauge tube; the water begins to boil under the reduced pressure, the vapour formed being carried off by the pump as it is produced. By regulating the supply of heat and the rate of flow of the pump, a steady condition can be obtained, under which the water continues to boil, and the pressure and temperature remain unchanged; the pump removes the vapour as fast as it is formed. Read the temperature and the height of the mercury column in the gauge tube. Subtract this latter from

the barometric height and thus obtain the pressure within the apparatus.

Alter the supply of gas and the rate at which the pump is working and thus obtain a series of readings of corresponding pressures and boiling temperatures.

The results may be entered in tabular form thus :

Pressure in mm. of mercury.	Temperatures in degrees Centigrade.	Pressure in mm. of mercury.	Temperature in degrees Centigrade.
30	29·3	235	70·1
55	40	350	79·7
100	50	525	90
150	60·2	760	100

The beaker at *C* is not *necessary* for the experiment ; it forms however a convenient addition, for without it water vapour finds its way into the space above the gauge column and condenses there ; in measuring the pressure the presence of this water has to be allowed for. Even with the beaker some vapour passes into the gauge and condenses ; this may either be allowed for in the calculations or it may be vaporized from time to time by heating the gauge tube gently. If a continuous acting air-pump be not available, an arrangement for condensing the vapour must be used. The glass tube from the retort *AB* is bent downwards and communicates with a large flask ; this flask is in communication with the air-pump. A three-necked bottle is convenient for this, the third opening being connected to the gauge. This bottle also serves the purpose of the beaker at *C* in fig. 55. The lower part of the tube from the retort *AB* is surrounded by a condenser jacket through which water at various temperatures can be made to flow. This water condenses the steam in the tube and the liquid formed by the condensation runs down into the bottle. The apparatus is partly exhausted by the air-pump. As the water boils under reduced pressure the vapour formed is condensed and the pressure and temperature can be read as before.

122. Boiling-point of a liquid. A comparison of the Tables on pp. 130 and 141 shews that the pressure under which water can boil at a given temperature is its vapour pressure at that temperature. We thus obtain the following

Definition of the boiling-point of a liquid. *The boiling-point of a liquid is the temperature at which the*

vapour pressure of the liquid is equal to the pressure on the surface of the liquid.

The necessity for the correction described on p. 20 and the method of calculating it can now be explained.

The second fixed point on a thermometer registers the temperature of steam issuing from boiling water at a standard pressure, that due to 760 mm. of mercury; if the pressure be lower than this the temperature of the steam will be less than 100° C. Experiments such as have been described shew that water boils at a temperature of 99° C. if the pressure be 733.2 mm., and further that for small variations the change in boiling-point is very approximately proportional to the change in pressure.

Thus the boiling-point falls by 1° C. for a reduction of pressure of 26.8 mm., and the change in boiling-point caused by any not very large change of pressure is found by dividing that change by 26.8.

123. Boiling under increased pressure. In a similar manner if the pressure be increased the temperature of the boiling-point is raised. This can be shewn by boiling water in a strong metallic vessel fitted with a pressure gauge and a safety-valve. A thermometer is inserted with its bulb in the steam. It is found that at a pressure of about two atmospheres the boiling-point is raised to 120° C.

This fact is made use of in high-pressure engines to raise the temperature of the steam, which in some cases may be as high as 200° C.; the pressure of the steam will then be about 15 atmospheres.

***124. Measurement of heights by Boiling-points—Hypsometry.** By means of a Table such as that given on p. 141 two observations of the boiling-point may be used to determine the difference of level between two stations. For, by observing the boiling-point, the atmospheric pressure can be determined from the Table. Thus the difference of pressure between the two stations can be found. But this difference of pressure is the weight of a vertical column of air 1 sq. cm. in all between the two stations. Hence the mass of this column

is known; we require to find its height. Now the density of air depends on its pressure and temperature: if the stations are not too far apart, we may assume that the average pressure and temperature are the mean of those observed at the two stations. From this the average density of the air between the two can be obtained, since we know that the density of dry air at 0°C . and 760 mm. is $\cdot 001293$ gramme per c.cm. Calculate the average density of the air at the pressure and temperature of the observation and divide the mass of the column by this density; the result will give the height of the column, that is the difference in level between the two stations. The result is only approximately correct, for the assumptions made are only approximately true. A more elaborate calculation however can be made, from which a more accurate result can be found.

It can be shewn that the height in centimetres is approximately given by the following rule.

Multiply the difference between the logarithms of the two barometer readings by 2×10^6 , the result is the required height.

Example. *The boiling-point at the lower station is $99^{\circ}\cdot 7$ and at the upper station 95°C ., the temperature of the air at the two stations being 13°C . and 7°C . Find the difference in level.*

The pressure at the lower station is that due to 751·9 mm. of mercury and at the upper that due to 633·7 mm. The difference of pressure is 118·2 mm. and the mean pressure 692·8 mm. and the mean temperature 10°C .

The mass of 1 c.cm. of mercury being 13·59 grammes, the mass of the column of air 1 sq. cm. in area between the stations is therefore

$$13\cdot 59 \times 11\cdot 82 \text{ grammes.}$$

The density of air at 10°C . and 692·8 mm. is

$$\frac{\cdot 001293 \times 692\cdot 8 \times 273}{760 \times 283} \text{ grammes per c.cm.,}$$

and hence the difference in level is

$$\frac{13\cdot 59 \times 11\cdot 82 \times 760 \times 283}{\cdot 001293 \times 692\cdot 8 \times 273} \text{ cm.,}$$

or about 1413 metres.

In obtaining this result no allowance has been made for the aqueous vapour in the air. In consequence of its presence the density should be rather greater than the value used in the calculations, and the height therefore rather less.

125. Heat required for evaporation. We have already seen in the Sections on Calorimetry that heat is needed to produce evaporation, and have explained the method of finding its amount in the case of water boiling at the ordinary pressure; the latent heat of evaporation has been defined as the quantity of heat required to change one gramme of liquid to vapour without change of temperature. It is in consequence of this fact that a body can be cooled by rapid evaporation. Thus drop a few drops of ether on the hand, it immediately feels cooler; or wrap a little muslin round the bulb of a thermometer and sprinkle a little ether over the muslin; the temperature as indicated by the thermometer falls, for the ether evaporates, absorbing heat from the mercury and cooling it. Energy is changed from the form of heat to that of the agitation of the molecules of the vapour.

EXPERIMENT (39). *To freeze water by rapid evaporation.*

Pour a little water on to a block of wood; take a small thin metal capsule and place it in the water. Pour a little ether into the capsule and then cause the ether to evaporate rapidly by blowing at it with a small pair of bellows. Heat is absorbed from the water which is cooled and finally frozen.

126. The Cryophorus. The action of this instrument affords another instance of the production of a low temperature by rapid evaporation. It consists of two glass bulbs *A* and *B*, connected by a tube. There is some water in one of the

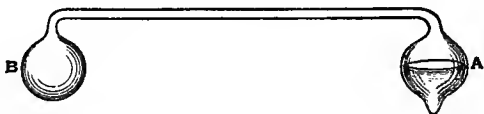


Fig. 56.

bulbs and the whole has been freed from air by boiling before the bulb was finally sealed up; thus the instrument contains nothing but water and the vapour of water. Get all the water into the bulb *A* and place it inside a beaker, covering

it with cotton-wool wadding or some non-conducting material¹.

Now surround the second bulb, *B*, with a freezing mixture carefully made; the vapour which the bulb contains is condensed and frozen; this reduces the pressure on the water in *A*, more vapour is given off, absorbing heat from the water, which is thereby cooled, and this process goes on so rapidly that the water in *A* is cooled down to the freezing-point, and finally frozen. Vapour is produced in *A* and passes over to *B*, carrying with it its heat which it gives up to the freezing mixture. Some of this is warmed by the energy abstracted from *A* and the water in *A* thereby is frozen.

127. Practical consequences. Freezing Machines. The evaporation of liquids is commonly used as a means of producing very low temperatures; various freezing machines depend on it for their action.

Atmospheric air and oxygen have recently been liquefied in large quantities by Prof. Dewar at the low temperature produced by the evaporation of ethylene, and remain liquid at atmospheric pressure in consequence of the low temperature produced by their own very rapid evaporation.

If a quantity of carbonic acid be liquefied under pressure and then allowed to escape as a fine jet, the evaporation from the outer surface cools the core of the jet so that the liquid is frozen and can be collected as a solid.

To do this the jet of liquid carbonic acid is allowed to escape into a closed wooden box. On opening the box, solid carbonic acid is obtained; the solid melts but slowly, when exposed to the air, for it takes time to absorb the heat necessary to melt it; but, by pouring sulphuric ether on the mass, melting is facilitated and an intense freezing mixture is produced by which mercury can be frozen readily. Pour a little mercury into a hollow in a block of wood and place a

¹ If this be not done the water in the bulb when cooled absorbs heat rapidly from the air around, and the experiment may fail in consequence. If the room be very warm, it is sometimes desirable to cool the instrument by putting it in cold water before beginning.

wire in the mercury. Place some solid carbonic acid on the mercury and then pour some ether over the whole. The mercury is frozen solid and can be lifted off the block by the wire. Immerse the solid mercury in water ; it melts, and each drop as it falls downwards through the water freezes the water with which it comes in contact, forming tubes of ice through which the liquid mercury pours.

128. Hygrometry. The formation of dew. There is generally some aqueous vapour present in the air. This may be shewn by various experiments. Thus, place a small quantity of calcium chloride in a saucer and leave it in the room. The chloride gradually dissolves in the moisture it absorbs from the air. Or again, dry carefully the outside of a flask or beaker, and then fill it with very cold water, the outside of the flask becomes dimmed with moisture deposited on it from the air.

Fill the lower part of a U tube with fragments of pumice which have been soaked in sulphuric acid, then heated to redness, and finally saturated with the acid. Close each end with a cork, through which a narrow glass tube passes, and weigh the tube and its contents carefully. Attach one end of the tube to an air-pump, and draw the air of the room through it for a time, then weigh again ; it will be found that the tube has increased in weight. This increase is due to the aqueous vapour absorbed from the air by the acid. The condition of the atmosphere with regard to the aqueous vapour present in it is spoken of as its hygrometric state, and the part of Physics which deals with this is called Hygrometry.

EXPERIMENT (40). *To measure the mass of aqueous vapour present in a given volume of air and to find the pressure due to the vapour.*

This is done by passing a known volume of air through drying tubes and measuring their increase in mass. Take a large bottle such as a Winchester quart or a small carboy. Find its volume by filling it with water from a measuring flask. Bore three holes in a cork which will close its mouth ; fit a thermometer into one, and through the others pass two glass tubes which reach down to the bottom of the vessel. Bend

one of these tubes to form a siphon by which the bottle can be emptied. On emptying the bottle by this siphon air is drawn in through the other tube, and, when the bottle is completely emptied, a known volume of air has entered. Fill the bottle and siphon and close the end with an india-rubber tube and a pinchcock. The second tube is connected to a small bottle, *B*, filled with freshly fused calcium chloride.

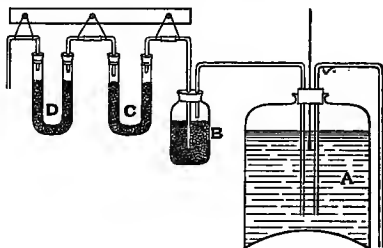


Fig. 57.

Another glass tube passing to the bottom of this bottle is connected with the drying tubes. Two of these, *C*, *D*, placed one behind the other, are used. When air passes through, most of its moisture is absorbed by the first drying tube; the increase of weight therefore of the second tube *C* should be very small. The object of the bottle *B* is to prevent the moisture from the aspirator *A* reaching the drying tubes, all that passes along the tube between *A* and *B* is absorbed by the calcium chloride in the bottle.

Weigh the drying tubes and connect up the apparatus, using very short pieces of india-rubber tubing to make the connections¹.

Since all the air which enters the aspirator is supposed to pass through the drying tubes, it is necessary that the connections should be air-tight. To test this close the tube at

¹ This india-rubber tubing may absorb some moisture. A better way therefore of making the connections is that described in Glazebrook and Shaw's *Practical Physics*, Section 42, but it needs somewhat more elaborate apparatus.

the end of the drying tubes with a pinchcock; on opening the siphon tube no water should run out; if this condition holds, remove the pinchcocks and allow the water to flow from the aspirator. Read the temperature from time to time. When the aspirator is empty disconnect the drying tubes and weigh again; the increase in weight gives the mass of aqueous vapour which was contained in the volume of air, which, at the temperature and pressure of the experiment, fills the aspirator.

The mass of aqueous vapour per c.cm. of air under the conditions of the experiment is thus found.

We can find from this the pressure due to this aqueous vapour thus.

The mass of a cubic centimetre of dry air at freezing-point and at a pressure of 760 mm. is known to be $\cdot 001293$ gramme. Let e be the pressure of the aqueous vapour, and T the absolute temperature¹ at the time of the experiment. We wish to determine e . Now the mass of air which at pressure e and temperature T would occupy 1 c.cm. is²

$$\cdot 001293 \times \frac{e}{760} \times \frac{273}{T}.$$

Moreover it is known that the specific gravity of aqueous vapour referred to air at the same pressure and temperature is $\cdot 622$. Hence the mass of aqueous vapour which at pressure e and temperature T occupies 1 c.cm. is

$$\cdot 622 \times \cdot 001293 \times \frac{e}{760} \times \frac{273}{T} \text{ grammes,}$$

and it is this mass of aqueous vapour which has been determined by the experiment. Let us suppose the result of the experiment to have shewn that there are w grammes of vapour per c.cm. of air. Then

$$w = \frac{\cdot 622 \times \cdot 001293 \times e \times 273}{760 \times T},$$

and from this on reducing we find

$$e = 3460 \times T \times w \text{ mm.}$$

The quantity w is measured it must be remembered in grammes per c.cm.

In finding w from the observations a correction is required because the air in the aspirator is saturated while that outside is not; the effect of this will however be small. See Glazebrook and Shaw, *Practical Physics*, § 42.

¹ See § 101. $T = 273 + t$, if t is temperature Centigrade.

² See § 102.

129. The Dew-point. Under ordinary circumstances, the air is not saturated with aqueous vapour, so that the pressure exerted by the vapour is less than the saturation pressure. If the air in a room or open space be cooled down, the pressure remains constant and equal to the atmospheric pressure; the air contracts in volume and more air enters from outside, but the pressure does not change. For the pressure of the aqueous vapour in the air, this statement again is true, supposing the air not to be saturated. This pressure also remains constant until, as the cooling proceeds, a temperature is reached at which the air becomes saturated; the pressure of the vapour at this temperature is the same as it was originally; but, if the air be cooled below this temperature, some of the vapour is condensed as moisture and the pressure falls. The temperature at which such condensation takes place is called the dew-point, and the saturation pressure of the aqueous vapour at the dew-point is equal to the pressure of the aqueous vapour under the original conditions.

Definition. *The temperature at which a mass of air is saturated with the aqueous vapour it contains is called the dew-point.*

It follows from the above that the pressure of the aqueous vapour in the air, under given conditions, is equal to the saturation pressure at the dew-point. Thus we can find the pressure of the aqueous vapour present by determining the dew-point, and then finding from the Table on p. 130, the saturation pressure at that temperature. There are various methods of finding the dew-point: these will be described shortly.

130. Relative humidity. The feeling of wetness or dampness in the air does not depend mainly on the absolute amount of vapour present but rather on the nearness of that vapour to its point of saturation. Thus, on a warm day in summer, there is probably much more vapour present per cubic centimetre than on a damp winter day. In the latter case, however, the temperature of the air is very little above the dew-point, the air is nearly saturated. A very small fall of temperature will cause the vapour to be deposited as moisture.

On the summer day however, though much more moisture is present, and the dew-point is therefore higher, the temperature is much higher; a greater fall is needed before the dew-point is reached and the air begins to feel damp.

Definition. *The ratio of the pressure of the aqueous vapour present in the air at a given temperature to the saturation pressure at that temperature is called the relative humidity.*

Thus, to determine the relative humidity we require to find the dew-point and to make use of Regnault's Table of saturation pressures.

Example. *The temperature of the air is 16° C., the dew-point is 10° C.; find the relative humidity.*

The pressure of the aqueous vapour present is equal to the saturation pressure at the dew-point or 10°, and this is 9·17 mm. The saturation pressure at 16° is 13·54 mm.

Thus the relative humidity

$$= \frac{9\cdot17}{13\cdot54} = \cdot677.$$

131. Determination of the Dew-point.

EXPERIMENT (41). *To determine the dew-point and to find the pressure of the aqueous vapour present in the air.*

Take a small beaker or wide test tube, coat the inside with Brunswick black, and allow it to harden. Fit the vessel with a stirrer, a piece of bent copper wire serves for this, and fill it about three-quarters full with water at about the temperature of the room. Add some ice in small pieces, stirring well, and waiting till each piece is melted before adding the next. Keep a thermometer in the water during the process, and watch the blackened surface of the glass carefully. The temperature of the surface falls and, after a time, reaches the temperature of the dew-point. Moisture begins to be deposited from the air on the glass, and this deposit is fairly easily seen on the blackened surface. Read the temperature at which the deposition begins. This temperature is the dew-point. Look up in the Table p. 130 the saturation pressure of aqueous vapour which corresponds

to this temperature. This pressure is the pressure of the aqueous vapour present in the air at the time of the experiment.

The results obtained by this rough apparatus are liable to various errors. It is not easy to determine accurately the temperature at which the dew begins to be deposited, and various forms of hygrometers have been devised with a view to its more exact measurement.

132. Daniell's Hygrometer. This consists of two glass bulbs, *A* and *B* (fig. 58), connected by a tube. The two bulbs contain ether and ether vapour, the air having been exhausted from them before the apparatus was sealed up. The one bulb, *A*, is made of blackened glass and inside it there is a thermometer, the bulb of which is in the ether, while the scale is visible above the blackened bulb. The thermometer gives the temperature of the ether. The other bulb *B* which contains only ether vapour when the experiment begins is covered with some muslin.

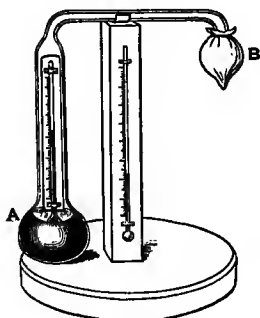


Fig. 58.

A second thermometer mounted outside gives the temperature of the air.

To perform an experiment pour a little ether on the muslin covering of the bulb *B*. The ether evaporates, absorbing heat in the process and thus cooling down the vapour in the bulb. Some of the vapour is condensed, and in consequence more ether evaporates in *A*. The bulb *A* is thereby slowly cooled down, until it reaches the temperature of the dew-point, which is indicated by a deposit of dew on the outside of the bulb. Read the temperature as given by the thermometer in *A*. This temperature will be the dew-point provided (1) that the first deposit of dew has been observed, and (2) that the temperature of the ether is also the temperature of the outside of the glass bulb. As a matter of

fact it is probable that neither of these conditions has been satisfied: the error introduced may to some extent be compensated for by stopping the experiment as soon as dew is observed, and allowing the apparatus to rise in temperature by absorbing heat from the room. Watch for the disappearance of the dew and note the temperature at which this takes place. This temperature should not differ greatly from that at which the dew appeared; and the mean of the two will give a more accurate value for the dew-point.

133. Regnault's Hygrometer. This consists of a thin silver tube, fig. 59, like a short test tube mounted on a stand. Two small tubes *A* and *B* (fig. 59 *a*) enter the larger tube, one of them *B* opening near the bottom, the other *A* near the top. A delicate thermometer passes through a cork which closes the silver tube, the bulb of the thermometer, which should be small, being near the bottom. The test tube

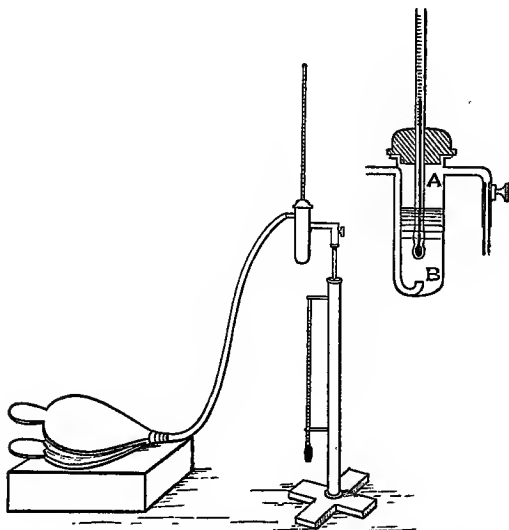


Fig. 59.

Fig. 59 a.

is partly filled with ether, covering the thermometer bulb, and air is caused to bubble through the ether either by blowing it in with a pair of bellows through *B*, or sucking it out with a pump, through *A*. This produces evaporation of the ether and a consequent fall of temperature, until, at last, the ether and tube are reduced to the dew-point. This is indicated by the deposit of dew on the silver tube. Observe the temperature at which this deposit takes place. Allow the evaporation to stop and the temperature to rise until the dew disappears. Note the temperature at which this occurs; it should not differ greatly from that first observed, and the mean of the two may be taken as the dew-point.

134. Dines' Hygrometer. In this instrument a delicate thermometer, *DE*, is fixed in a horizontal position in a groove in a wooden block.

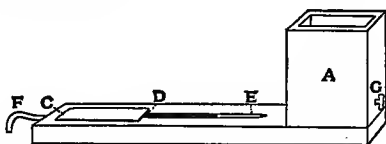


Fig. 60.

The bulb is placed inside a metal box, the upper side of the box being covered by a very thin piece of black glass, and the thermometer bulb being in contact with the glass.

A tube leads from the box, *B*, to a reservoir, *A* (fig. 60), and is closed near the reservoir by a tap *G*. An overflow tube *F* leads out of the box. The reservoir is filled with water



Fig. 60 a.

cooled by ice. This cooled water is allowed, by adjusting the tap, to pass slowly along the tube through the box. The glass plate is thus cooled down, its temperature being given by the thermometer, until a deposit of dew becomes visible. As soon as this is seen, the temperature is read, and the flow stopped. The temperature at which the dew disappears is also observed, and the mean of the two when they do not differ greatly may be taken as the dew-point.

When the temperature of the dew-point has been determined the pressure of the vapour can be found from the Tables as has been already described. When this pressure e is known the formula proved in § 128 will give w the number of grammes of aqueous vapour in a cubic centimetre of air; for we have, if T is the absolute temperature,

$$w = \frac{e}{3460 \times T}.$$

135. The wet and dry bulb Thermometers. This instrument, which consists of two thermometers mounted side by side, is often used to determine the pressure of the aqueous vapour in the air (fig. 61). The bulb of one of the two thermometers, A , is kept constantly moist by means of a piece of cotton-wick wrapped round it, and dipping into a small vessel of water at the side. This wick should be thoroughly cleansed from grease by boiling in caustic potash, and then washing with distilled water; this will ensure a flow of water to the bulb.

The water evaporates, and absorbs heat from the thermometer. Thus the temperature as indicated by this thermometer is lower than that given by the thermometer B . The difference will depend on the rapidity with which the evaporation proceeds, and the rate of evaporation will depend on the temperature and the amount of moisture already present.

If the air is very dry, evaporation will proceed rapidly, heat will be absorbed from the thermometer at a rapid rate and the temperature of A will fall considerably below that of B . If, on the other hand, the air be nearly saturated, evaporation will be slow and the difference of temperature small.

A formula can be obtained, connecting together the pressure of the aqueous vapour, the two temperatures, and some other quantities which can be observed. From this

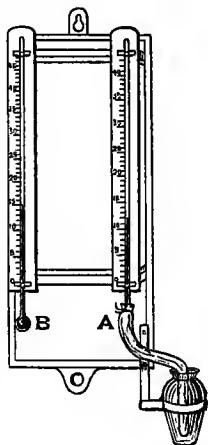


Fig. 61.

formula, which can be verified by direct experiment, Tables have been constructed giving the pressure of the aqueous vapour, when the temperature of the dry-bulb thermometer is observed, and also the difference of temperature between it and the wet-bulb thermometer.

Such a Table is given below, in which the first vertical column gives the readings of the dry bulb thermometer and the top horizontal column the difference of temperature in degrees centigrade between the dry and the wet bulb thermometers. The Table shews the pressure in mm. of mercury of the vapour present in the air. And from the first two columns of the Table the dew-point can be found.

t°C.	0	1	2	3	4	5	6	7	8	9	10	11
0	4.6	3.7	2.9	2.1	1.3							
1	4.9	4.0	3.2	2.4	1.6	0.8						
2	5.3	4.4	3.4	2.7	1.9	1.0						
3	5.7	4.7	3.7	2.8	2.2	1.3						
4	6.1	5.1	4.1	3.2	2.4	1.6	0.8					
5	6.5	5.5	4.5	3.5	2.6	1.8	1.0					
6	7.0	5.9	4.9	3.9	2.9	2.0	1.1					
7	7.5	6.4	5.3	4.3	3.3	2.3	1.4	0.4				
8	8.0	6.9	5.8	4.7	3.7	2.7	1.7	0.8				
9	8.6	7.4	6.3	5.2	4.1	3.1	2.1	1.1	0.2			
10	9.2	8.0	6.8	5.7	4.6	3.5	2.5	1.5	0.5			
11	9.8	8.6	7.4	6.2	5.1	4.0	2.9	1.9	0.9			
12	10.5	9.2	8.0	6.8	5.6	4.5	3.4	2.3	1.3			
13	11.2	9.8	8.6	7.3	6.2	5.0	3.9	2.8	1.7			
14	11.9	10.6	9.2	8.0	6.7	5.6	4.4	3.3	2.2	1.1		
15	12.7	11.3	9.9	8.6	7.4	6.1	5.0	3.8	2.7	1.6	0.5	
16	13.5	12.1	10.7	9.3	8.0	6.8	5.5	4.3	3.2	2.1	1.0	
17	14.4	13.0	11.5	10.1	8.7	7.4	6.2	4.9	3.7	2.6	1.5	0.4
18	15.4	13.8	12.3	10.9	9.5	8.1	6.8	5.5	4.3	3.1	2.0	0.9
19	16.4	14.7	13.2	11.7	10.3	8.9	7.5	6.2	4.9	3.7	2.5	1.4
20	17.4	15.7	14.1	12.6	11.1	9.7	8.3	6.9	5.6	4.3	3.1	1.9
21	18.5	16.8	15.1	13.5	12.0	10.5	9.0	7.6	6.3	5.0	3.7	2.5
22	19.7	17.9	16.2	14.5	12.9	11.4	9.9	8.4	7.0	5.7	4.4	3.1
23	20.9	19.0	17.3	15.6	13.9	12.3	10.8	9.2	7.8	6.4	5.1	3.8
24	22.2	20.3	18.4	16.6	14.9	13.3	11.7	10.1	8.7	7.2	5.8	4.5
25	23.6	21.6	19.7	17.8	16.0	14.3	12.7	11.1	9.5	8.0	6.6	5.2
26	25.0	22.9	21.0	19.0	17.2	15.4	13.7	12.1	10.5	8.9	7.4	6.0
27	26.5	24.9	22.3	20.3	18.4	16.6	14.8	13.1	11.4	9.8	8.3	6.8
28	28.1	25.9	23.7	21.7	19.7	17.6	16.0	14.2	12.5	10.8	9.2	7.7
29	29.8	27.5	25.3	23.1	21.1	19.1	17.2	15.3	13.6	11.9	10.2	8.6
30	31.6	29.2	26.9	24.6	22.5	20.5	18.5	16.6	14.7	13.0	11.2	9.6

Example. *The temperature of the dry-bulb thermometer is 15° , that of the wet-bulb thermometer 12° ; find the pressure of the vapour and the dew-point.*

The difference between the two is 3° . Now the first column of the Table gives the temperature of the dry-bulb thermometer and the top line the differences. Look down the column headed 3° until you come to the number in the same horizontal line as the 15° in the first column. This is the pressure required; and from the Table it is found to be 8.6 mm. Now the Table on p. 130 shews that the saturation pressure at 9° is 8.57 mm., and the dew-point is the temperature at which the actual pressure is the saturation pressure. The dew-point is thus just over 9° . To find it more exactly observe from the Table that, for 1° , from 9° to 10° , the saturation pressure increases from 8.57 to 9.17 or through .6 mm. We wish to find the change in dew-point corresponding to an increase in pressure of .03 mm. ($8.6 - 8.57$). This change will be $.03/.6$ of 1° C. or $0^{\circ}.05$ C.

Thus the dew-point is $9^{\circ}.05$ C.

136. Simple Hygrometers. The action of many simple forms of hygrometers depends on the fact that some substances readily absorb aqueous vapour when it is present in the air, and change their form or appearance in the process.

A substance, which readily absorbs moisture, is said to be hygroscopic.

Thus sea-weed is very hygroscopic; a bunch of sea-weed hung up in the air, in damp weather, when there is plenty of vapour present, absorbs some of it and swells up considerably.

Catgut again is hygroscopic. If a piece of catgut be hung up, supported at one end, and carrying a light weight at the other, it will absorb moisture in damp weather and untwist; if the weather be dry it twists up again.

These changes are made use of in the "old man and his wife" hygroscope, in which one figure comes out in dry weather, the other in wet.

In de Saussure's hygroscope a light weight is hung at the end of a long hair which is hygroscopic. This end of the hair is attached to the short end of a lever. As the hair becomes damp, it contracts in length, and a small elongation or contraction is shewn by a considerable motion of the long end of the lever.

In other hygrometers some hygroscopic material is coated

with a substance such as a salt of cobalt which changes colour on the absorption of moisture. When there is much aqueous vapour present blue becomes red.

The method of treating numerous problems, involving hygrometry, and the pressure of vapours, may be best illustrated by the following numerical example.

Example. *Ten litres of oxygen are collected over water at a pressure of 750 mm. and a temperature of 15°, being saturated with vapour; find the mass of dry oxygen and of aqueous vapour, having given that the density of hydrogen at 0° and 760 is .0896 gramme per litre, and that the specific gravities of oxygen and of aqueous vapour, referred to hydrogen, are 16 and 9 respectively.*

It follows from this that the mass of a litre of oxygen under standard conditions is $16 \times .0896$ or 1.434 gramme, while that of a litre of aqueous vapour under the same conditions is .806 gramme. The total pressure of the damp oxygen is the sum of the pressure due to the oxygen and the pressure due to the vapour. This latter, since the space is saturated at 15°, is 12.7 mm. Thus the pressure due to the oxygen is $750 - 12.7$, or 737.3 mm. We have thus 10 litres oxygen at 15° and 737.3 mm.

The volume of this at 0° and 760 will be

$$\frac{10 \times 737.3 \times 273}{760 \times 288} \text{ litres,}$$

and the mass
$$\frac{10 \times 737.3 \times 273 \times 1.434}{760 \times 288} \text{ grammes.}$$

This reduces to 13.19 grammes, while since there are 10 litres of vapour at 15° and 12.7 its volume at 0° and 760 will be

$$\frac{10 \times 12.7 \times 273}{760 \times 288} \text{ litres,}$$

and its mass
$$\frac{10 \times 12.7 \times 273 \times .806}{760 \times 288} \text{ grammes,}$$

which reduces to .128 gramme.

EXAMPLES.

VAPOURS AND HYGROMETRY.

1. One hundred cubic centimetres of oxygen, saturated with aqueous vapour, are collected at a pressure of 740 mm. and a temperature of 15° C. Find the volume of dry oxygen at 0° and 760 mm., having given that the maximum pressure of aqueous vapour at 15° is 12.7 mm.

2. Distinguish between saturated and unsaturated vapours.

What is meant by the statement, that when the dew-point is $20^{\circ}\text{C}.$, the maximum pressure of aqueous vapour in the air is that due to 17.4 mm. of mercury?

3. Distinguish between a vapour and a gas. The pressure of aqueous vapour at a temperature of $50^{\circ}\text{C}.$ is .12 atmospheres; how would you proceed to verify this statement?

4. How would you distinguish between *vaporisation* and *ebullition*? Does the boiling-point of a liquid depend on the pressure on its surface? Illustrate your answer with an experiment.

5. What is Dalton's law as to the pressure of vapours? How is it used in finding, from observations of the dew-point, the pressure of the aqueous vapour in the air?

6. Describe an experiment showing that water can be frozen by its own evaporation. What weight of vapour must evaporate in order to freeze a gramme of water already at freezing-point?

7. Explain how to determine the amount of aqueous vapour present in the air from a knowledge of the dew-point.

8. Define the dew-point. How is dew formed, and why is it more copious on some substances than on others?

9. Calculate the weight of a litre of hydrogen collected over water at 15° when the height of the barometer is 765 mm.; the density of hydrogen at 0° and 760 mm. is .000089 gramme per c.cm., and of aqueous vapour nine times that of hydrogen, while the maximum pressure of aqueous vapour at 15° is 12.7 mm.

10. In a chemical experiment a certain quantity of oxygen is given off, and this is collected over water in a tube graduated in c.cm.: the volume read off on the tube is 40 c.cm. and the water stands in the tube 50 centimetres above the water outside the tube: it is required to find the mass of the gas.

The water is at a temperature of $20^{\circ}\text{C}.$ and the barometer reads 755; the density of mercury is 13.96 grammes per c.cm.: the pressure of aqueous vapour at 20° is 17.4 mm. of mercury: and the mass of a cubic centimetre of oxygen at $0^{\circ}\text{C}.$ and 760 mm. pressure is .00143 gramme.

11. Explain the following table taken from the weather report of the "Times."

	Temperature		Pressure of vapour. Inches	Weight of vapour in 10 cubic feet of air	Drying power of air per 10 c. ft.	Humidity Saturation=100
	Air	Dew-point				
Noon	67°	38°	.229	25 grains	48	34
9 p.m.	58°	38°	.229	26 "	28	48
2 a.m.	54°	38°	.229	26 "	21	55

12. A closed vessel contains air saturated with water vapour at a temperature of 100°C . When the temperature is raised to 150°C . without alteration of volume the pressure is that due to 2 atmospheres. What would be the pressure of the air alone at 0°C ., when occupying the same volume?

13. State and explain the law as to the pressure of a mixture of air and water vapour. What is meant by the hygrometric state of the air? Explain how to determine the pressure of the aqueous vapour in the air from a knowledge of the dew-point and a table of the saturation pressures of aqueous vapour.

14. The temperature of the air in a closed space is observed to be 15°C . and the dew-point 8°C . If the temperature falls to 10°C . how is (1) the dew-point, (2) the pressure of aqueous vapour in the air affected?

15. The dew-point is observed to be 15° , and the temperature of the air is 20° . The saturation pressure of aqueous vapour at 15° is 12.7 mm., find the pressure of the vapour in the air.

16. Find the weight of a litre of hydrogen and water vapour at 15°C . and under a pressure of 750 mm. of mercury. Of this pressure 18 mm. is due to the water vapour, the rest being due to the hydrogen. A litre of hydrogen at 0°C . and at a pressure of 760 mm. weighs $\frac{1}{14}$ of a gramme.

17. Find the weight of a cubic metre of air saturated with moisture, the pressure being 750 mm. of mercury, and the temperature 15°C .

(Pressure of aqueous vapour at 15°C . = 12.7 mm. of mercury, weight of 1 c.cm. of dry air at 0°C . and 760 mm. = .001293 gr. Specific gravity of aqueous vapour referred to air = .623.)

18. Why is it necessary to note the height of the barometer when determining the upper fixed point of a thermometer? How is this determination made? An alteration of pressure of 26 mm. alters the boiling-point of water 1° . Find the boiling-point when the height of the barometer is 745 mm.

19. Describe and explain the action of the Cryophorus. Water can be frozen in a dry climate by exposing it in shallow pans under a clear sky even though the temperature of the air be above the freezing-point. Explain this.

CHAPTER X.

THE TRANSMISSION OF HEAT. CONDUCTION.

137. Conduction of Heat. If one end of a piece of metal be placed in the fire the other end gradually becomes hot.

Heat-Energy is communicated to the particles of metal in the fire. These communicate some of their energy to the neighbouring particles and thus the energy travels from particle to particle along the bar. This process is called the conduction of heat.

Definition. *Heat is said to be transmitted by conduction when it passes from the hotter to the colder parts of a body or from one body to a colder body in contact with it.*

138. Convection of Heat. The air over any hot surface has its temperature raised and in consequence expands and, becoming less dense than the surrounding air, rises, carrying its heat-energy with it. A process such as this is called convection. The heat is transmitted by the actual motion of the heated particles.

Definition. *Heat is transmitted by convection when material particles conveying the heat are carried from one point to another.*

139. Radiation of Heat. The sun's rays pass through the air without appreciably heating it. If they be allowed to fall on the bulb of a thermometer the temperature as

indicated by the thermometer rises; if the bulb be covered with lampblack the rise of temperature is greater than if the bulb be of clear glass. Energy reaching the thermometer from the sun has travelled through the air without greatly raising its temperature, and has been transformed into heat on reaching the thermometer bulb.

Experiment further shews that the energy in this case travels in straight lines or rays. It is known as radiant energy, and the process of its transference is called radiation.

There are some substances through which radiant energy passes unchanged in amount, in the case of other substances some or all of the radiant energy is transformed into heat; such substances are said to absorb the radiation they receive and their temperature is raised by this absorption.

Definition. *Heat-energy is transmitted by radiation when it passes from one point to another without raising the temperature of the medium through which it travels.*

140. Experiments on the Conduction of Heat.

If we touch various substances lying side by side in a room some will appear warmer than others even though we may find, on testing them with a thermometer, that their temperatures are the same; if the room be cold any pieces of metal in it will appear very cold to the touch, while the woollen curtains or carpet will seem to be quite warm. On the other hand if the temperature of the various objects be above that of the hand, the metallic bodies will appear distinctly warmer than the woollen.

The sensation of warmth depends not merely on the temperature of the objects touched but on the rate at which heat is transferred through their masses; the metallic bodies appear cold in the first case because heat can pass rapidly from the observer's hand into their interiors; they appear warm in the second case because heat passes rapidly from them to the observer.

In the case of a woollen body heat travels much more slowly and it does not appear so cold when touched, for the parts near the point of contact soon become raised to the

temperature of the hand and the gradual diffusion of heat into the interior is a slow process.

Thus these observations shew us that different bodies have different conductivities for heat; we shall define later the exact meaning of the term conductivity. The following experiments will illustrate this difference.

EXPERIMENT (42). *To shew that different materials conduct heat differently.*

(a) Take a number of rods of different materials, wood, glass, bone, copper, iron etc.; place one end of each in a beaker of boiling water. The exposed ends of the wood and glass rods are hardly appreciably raised in temperature; while the iron and copper rods become very hot.

(b) Take two rods, one of iron the other of copper, each about 30 cm. long and 5 mm. in diameter. Place one end of each rod in a Bunsen flame and, after waiting for some time for the temperature to become steady, find the point on each rod at which a fragment of paraffin wax placed on the rod is just melted. The temperature is the same at these two points; it will be found that the point on the copper rod is much further from the source of heat than that on the iron rod. In a given time more heat is transmitted along the copper than along the iron rod, and so the temperature rises to the melting point of paraffin at a greater distance from the source of heat.

(c) In fig. 62 *ABC* is a metal trough through which a

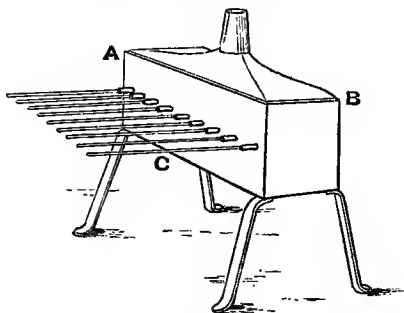


Fig. 62.

number of narrow tubes some 3 mm. in diameter pass. A number of rods of different materials are all covered with a thin coating of paraffin wax and then inserted in the respective tubes. The trough is filled with boiling water so that one end of each of the rods is raised to a high temperature. Heat is conducted down each of the rods, melting the wax as the temperature rises to the melting-point of the paraffin. It will be observed that the melting of the wax proceeds at a very different rate and extends to different distances along the bars. If the temperature of the trough be kept steady at 100° by passing in steam or by some other suitable means, it will be found that after a time the line of separation between the melted and unmelted parts of the wax no longer moves along the bar but remains stationary. This stationary position on each bar marks the points at which the temperatures of the respective bars are all equal to the melting-point of the wax, and it can be shewn that the conductivities of the bars are proportional to the squares of the distances of these points from the trough.

141. Definition of Thermal Conductivity. *Consider a plate of any material 1 cm. in thickness. Let one face be kept at a temperature of t° , the other at a temperature of $(t-1)^{\circ}$. The quantity of heat which passes per second through each unit of area of the plate is called the thermal conductivity of the material of the plate.*

We may calculate the quantity of heat passing through a plate of known thermal conductivity k thus.

Consider a plate of unit thickness: let the temperature of one face be t and of the other t' . Then experiment shews that for moderate differences of temperature the flow of heat per unit area is proportional to $(t-t')$, and is equal to $k(t-t')$ units per second.

Let the thickness of the plate be d cm., the temperature of the faces being t and t' , then in the steady state the fall of temperature is uniform through the plate, and since in d cm. the fall is $t-t'$ the fall per cm. is $(t-t')/d$. Hence the flow of heat per second per unit area of the plate is $k(t-t')/d$. If now the plate contains S sq. cm., since the flow for each unit

of area is the same, the total quantity of heat crossing is $k(t-t')S/d$ per second; and hence in T seconds the quantity Q which passes is given by

$$Q = \frac{k(t-t')S \cdot T}{d}.$$

Example. *The thermal conductivity of iron is .2. Find the quantity of heat passing per hour into a boiler the plates of which are 1.2 cm. thick, and the area of the heating surface 50000 sq. cm., the temperature of the outer surface being 120°, that of the inner 100°; find also the mass of water evaporated.*

The fall of temperature for 1.2 cm. is 20°.

Thus the fall per cm. is $\frac{20^\circ}{1.2}$.

The flow of heat per cm. per sec. is $\frac{20 \times .2}{1.2}$.

Hence the total flow per hour is $\frac{20 \times 2 \times 50000 \times 3600}{12}$ or 600,000,000 units. Hence taking the latent heat of water as 536 we have as the mass of water vaporised

$$\frac{600,000,000}{536} \text{ grammes,}$$

and this reduces to 1,119,500 grammes approximately.

142. Difference between thermal conductivity and rate of rise of temperature. When heat is applied to one end of a rod or bar the rate at which the temperature rises at any point does not depend only on the thermal conductivity, but also on the specific heat. The quantity of heat reaching the portion of the bar about the point in question will depend on the thermal conductivity only.

The rise of temperature produced in a unit of volume is proportional to the quantity of heat reaching the volume divided by the product of the density and the specific heat.

Thus the time which the bar at any given point will take to rise to a given temperature will be approximately proportional to the product of the density and the specific heat divided by the conductivity. Suppose now we take two bars of the same size of lead and iron respectively, the thermal conductivity of iron is about .16, that of lead .11, the product

of the specific heat and density for iron is 8·7 and for lead 3·6. Hence the times of rising to the same temperature for iron and lead are respectively in the ratio of 87/16 to 36/11 or as 5 to 3.

Thus when the bars are each first heated at one end, a point on the lead close to the source rises in temperature nearly twice as fast as a point at the same distance from the hot end on the iron, and hence a given temperature such as that required to melt wax is attained by a point on the lead bar more quickly than by the corresponding point on the iron bar.

This may be shewn by the following experiments.

EXPERIMENT (43). *To shew the difference in the rate of rise of temperature of lead and iron bars.*

(a) Take two short cylinders of lead and iron some 3 or 4 cm. in length and about 2 cm. in diameter with flat ends. Coat one end of each with wax and place them upright on the flat top of a metal box through which steam¹ is passing from a boiler. The lower ends of the cylinders become raised to 100°. Heat is conducted through the cylinders to the wax and it will be found that the wax begins to melt first on the lead.

(b) Take two rods of the same size, about 1 cm. in diameter and 15 or 20 cm. long, and solder them to opposite faces of a box through which steam can be passed, so that each projects in a horizontal direction as shewn in fig. 63. Coat each rod

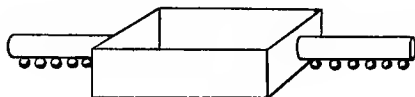


Fig. 63.

with a thin layer of paraffin and fasten, at intervals of about 1 cm., to the under side of each a number of shot about 3 mm. in diameter, securing them by imbedding them slightly in the

¹ A small biscuit box with the lid soldered on having suitable inlet and outlet tubes serves the purpose.

wax. Pass steam through the box allowing it to flow for some time, or if more convenient heat it by pouring in boiling water. The wax begins to melt and the shot to drop off first on the lead bar, but in the end, when the temperatures have become steady, the wax has been melted to a greater distance on the iron bar than on the lead, and more shot have dropped off it. The distance to which the wax is melted depends on the conductivity of the material, and this is greater for iron than for lead.

143. The variable and the steady state in the Conduction of Heat.

Consider any length AB of a bar such as that used in the last experiment. Suppose the temperature is measured at a number of points P_1, P_2 etc. along the bar and at each such point erect perpendiculars to the bar P_1Q_1, P_2Q_2 etc. to represent the observed temperatures.

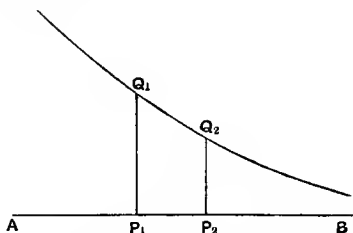


Fig. 64.

The ends of these perpendiculars will lie on a curve such as that shewn in fig. 64, and the temperature at any other point may be found by drawing a line perpendicular to the bar to meet the curve. Thus the change of temperature per centimetre length of the bar can be found at all points. This is called the temperature gradient; the quantity of heat entering any cross section is obtained by multiplying the temperature gradient by the area of the section and by the thermal conductivity. If we take sections at two points P_1 and P_2 the quantity of heat entering per second at P_1 is greater than that leaving at P_2 . If the temperature of the bar is changing the difference between the two quantities of heat is used partly in changing the temperature of the portion of the bar between P_1 and P_2 , partly in supplying the loss from the surface of this portion to the surrounding air. The conditions of the bar are then variable.

After a time a condition is reached in which the excess of the heat entering at P_1 over that leaving at P_2 is only just sufficient to supply the surface loss. When this state is

reached throughout the bar there is no further change of temperature, the state everywhere has become steady.

***144. Comparison of thermal conductivities.** EXPERIMENT (44). Take two long bars say of iron and copper from 1 to 2 metres long respectively. Bore a series of small holes in each at distances of 10 cm. apart, the holes being just large enough to contain the bulb of a thermometer; place a little mercury in each hole. Arrange the bars so that one end of each is inside a vessel which can contain melted lead or solder.

Such a vessel can conveniently be made out of a short length of iron pipe, the bars are secured by means of fireclay or some other luting into the opposite ends of the pipe. A large aperture cut in one side of the pipe serves to insert the lead or solder, and the whole is mounted with the bars in a horizontal position as shewn in fig. 65.

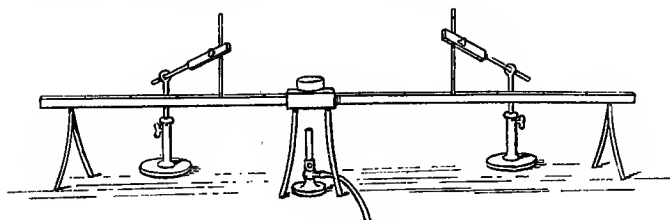


Fig. 65.

Heat the vessel in a Bunsen flame, to which, for accurate work, a gas regulator may be attached to keep the temperature constant.

When a steady condition has been reached place the thermometer in turn in each of the holes in the bars and read the temperatures. It will be found that the temperature falls much more rapidly along the iron, than along the copper bar. Take a number of points on the iron bar at which the temperatures are known, the holes in which the thermometers are inserted will be most convenient, and determine from the observations of the fall of temperature on the other bar the points which have the same temperatures as those selected on

the iron bar¹. Measure the distances of these points from the source of heat. The ratio of the squares of the distances of the respective points of equal temperature from the hot ends will be found to be the same, and this ratio can be shewn to be the ratio of the thermal conductivities.

***145. Measurement of thermal conductivity.** The amount of heat emitted per second from each centimetre of a bar such as one of those used in Experiment (44) when at a given temperature can be found from a separate experiment. This consists in heating the bar or a second similar bar to some uniform high temperature and leaving it to cool. The temperature is observed at equal intervals of time as the bar cools, and thus the rate of loss of temperature per second is found at various temperatures. From this the quantity of heat emitted per second by each centimetre when at a given temperature can be found.

Consider now a portion of the bar such as P_1P_2 in fig. 64: the quantity of heat entering per second at P_1 is kSa if S is the area of the bar and a the temperature gradient; the quantity leaving at P_2 is $kS\beta$, where β is the temperature gradient at P_2 , and thus we have when the steady state is reached

$kS(a - \beta)$ = heat emitted per second by the portion of the bar between P_1 and P_2 .

But since the temperature of the bar at all points between P_1 and P_2 is known, this quantity of heat emitted per second can be calculated from the results of the preliminary experiment, and then we have

$kS(a - \beta)$ = a known quantity of heat; and since S , a , β can all be observed, k is given by this equation.

This explains the principle of the method by which Forbes and others have measured the thermal conductivity of various substances.

The following is a Table of the values of the conductivities of various substances.

¹ This is best done by drawing a curve in which horizontal lines represent distances along the bar and vertical lines the observed temperatures at certain points, as described in Section 143.

Substance	Conductivity
Copper	·96
Iron	·20
Stone	·0059
Sand	·0026
Water	·0020
Fir-wood, across fibres	·00026
„ along fibres	·00047
Glass	·00050
Wool	·00012
Paraffin wax	·00014
Paper (unsized)	·000094
Air and some other gases	·000049

146. Practical effects of Conduction of Heat.

Other experiments have been devised to illustrate the difference in conductivity, and many domestic appliances depend for their action on the good or bad conductivity of various materials. Thus woollen materials such as blankets are used for warmth, for they are bad conductors and the heat of the body escapes slowly through them. For the same reason ice, which it is desired to keep, is wrapped in a blanket or placed in a box packed in sawdust; the heat of the room is conducted slowly through the sawdust.

A Norwegian cooking stove consists of a box packed in felt or some non-conducting material. The food which it is desired to cook is raised to the boiling-point and then the vessel which contains it is enclosed in the stove. The heat escapes so slowly that the contents are kept for some time at a temperature not greatly below boiling-point, and are cooked.

Water can be boiled in a vessel of thin paper without charring it, for the temperature of the water does not rise above 100° and if the paper be thin the heat applied to the outer surface is conducted through so rapidly that the temperature of that surface does not rise very greatly above 100° and so is not charred; if the thin paper be replaced by a vessel of cardboard, owing to the greater thickness of the cardboard the outer surface is raised considerably above the 100° C. and the cardboard is charred.

EXPERIMENT (45). *To illustrate the difference in conductivity between wood and metal.*

Take a cylinder about 3 or 4 cm. in diameter one half of which is wood, the other half metal. A piece of paper is wrapped round the cylinder and held over a Bunsen flame for a short time. The paper in contact with the wood soon becomes scorched and burnt, that in contact with the metal remains uncharred; the metal conducts the heat away so rapidly that the temperature never rises to the point at which the paper would be charred.

147. The Davy safety-lamp. The high conductivity of metals is made use of in the Davy lamp. To illustrate the principle of the lamp, take a piece of copper or brass gauze with a close mesh and lay it on the top of a tripod stand. Place a gas burner underneath in such a position that if the gauze be removed and the burner lighted the top of the flame would be above the former level of the gauze. Light the burner and replace the gauze. The gauze appears to damp out the flame, which only burns below; there is no flame above. Turn the gas off. When the gauze has cooled turn it on again and apply a light above the gauze. The gas now burns above but the light does not penetrate below. In both cases the heat is conducted away so quickly by the metal gauze, that the temperature of the inflammable gas on the side of the gauze remote from the flame is never raised to ignition point. In the first experiment some gas passes through unburnt and will ignite if a flame be applied above the gauze.

In the Davy safety-lamp shewn in fig. 66 the flame is surrounded by a thick wire gauze double in the part directly above the flame. If the lamp is brought into an atmosphere containing a little fire-damp (carburetted hydrogen) a blue cap shews itself above the flame. The temperature within is sufficient to ignite the fire-damp. If the blue cap increases in size, so as to fill or nearly fill the space within the gauze, danger is indicated, for

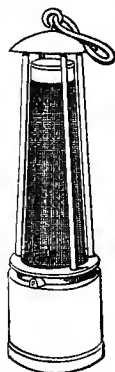


Fig. 66.

the temperature of the gauze may thereby be raised sufficiently to cause ignition outside. In more modern lamps many improvements have been introduced into the original form with a view of increasing the safety and improving the light.

148. Conduction of Heat in Liquids. Observations on the thermal conductivity of liquids are difficult to perform satisfactorily. Change of temperature produces variation in density, and convection currents (see Section 150) are set up which complicate the results. The following experiments however may be performed.

EXPERIMENT (46). *To illustrate the low thermal conductivity of water.*

(a) Arrange an air thermometer as shewn in fig. 67 so that the bulb can be surrounded by water.

This is most easily done by taking a small bell-jar with an open mouth, fitting to the opening a cork with a hole through which the tube of the thermometer can pass, and supporting the whole in a retort stand¹ with the bulb of the thermometer uppermost. Fill the bell-jar with water and arrange across its upper end a small tripod on which a capsule can rest.

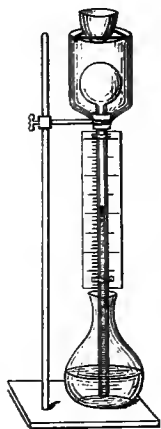


Fig. 67.

Place a small quantity of methylated spirits in the capsule and ignite it; the upper layers of water are heated, but it is some time before any effect is observed on the thermometer, and when a rise of temperature is noticed it will be found to be very slight indeed. In consequence of the low conductivity of the water the heat reaches the thermometer very slowly. On mixing the water in the bell-jar with a stirring rod a considerable rise of temperature is observed, shewing that the temperature of the upper layers can be raised by the heat applied without

¹ If a bell-jar is not readily obtainable, a wide-necked bottle from which the bottom has been removed will serve.

much affecting those below. If the water be replaced by mercury and the experiment repeated the rise of temperature will be much more rapid. Mercury is a good conductor of heat.

(b) Take a long narrow test tube and place a small lump of ice at the bottom, either weighting it so as to sink in water by wrapping some wire round it, or securing it at the bottom by a piece of wire gauze fitting the tube tightly.

Fill the tube with water and hold it with its upper part in the flame of a spirit lamp or Bunsen burner. The water becomes heated and may be boiled for some time without melting the ice. The heat is conducted down the tube very slowly.

149. Conduction of Heat in Gases. Air and other gases are very bad conductors of heat. The transference of heat through them takes place almost entirely by convection. The low conducting power of wool, feathers and sand and other porous substances is due in great measure to the air they contain. The material prevents the free circulation of the convection currents, the heat is transmitted in the main by conduction and the process is slow. If the bulb of an air thermometer as used in Experiment 46 be surrounded by a loose pile of powdered gypsum or some other light material and the experiment repeated, heat will reach it very slowly; if the powder be tightly packed round the bulb, the air being thereby expelled from the mass, the transference will be much more rapid.

When a pan or kettle containing water is placed on the fire the temperature of the under side of the pan does not rise much above 100° . At this temperature the gas given off by the coal does not ignite, hence there is under a great part of the pan a layer of comparatively cool gas. Such a layer conducts heat slowly and the process of boiling is thereby delayed. To obviate this the bottoms of some kettles are fitted with a number of small metal feet or legs. These project some way into the furnace, and their lower ends being at a considerable distance from the water can be raised in temperature much above boiling-point. The layer of non-conducting gas therefore is not formed over them and the heat is conducted to the water more rapidly.

CHAPTER XI.

TRANSMISSION OF HEAT BY CONVECTION.

150. Convection currents. When a liquid or gas is heated the density of the heated part becomes less, it therefore rises carrying its heat with it, and currents are set up in the substance by which the tendency toward uniformity of temperature is promoted.

We have had in Hope's experiment § 91 an example of such currents. Heat is transferred by the motion of the heated particles carrying their heat-energy with them. Such a process is called convection.

EXPERIMENT (47). *To illustrate the convection of heat.*

(a) Take a large sized flask and fill it with water, drop in some particles of aniline dye and heat it over a Bunsen burner. As the dye dissolves a stream of coloured water will be seen rising up the centre of the flask above the burner and diffusing gradually back down the sides where the water is cooler.

(b) Take a wide-mouthed bottle from which the bottom has been removed. Fit a cork to it and pass two glass tubes through the cork, one of these *AB* being straight, the other *CDE* bent, as shewn in fig. 68. Fit the other ends of the tubes through a cork in a flask in such a way that *AB* may just pass through the cork while

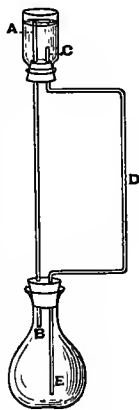


Fig. 68.

DE reaches nearly to the bottom. Secure the apparatus in a stand with *AB* vertical and fill it with water to above the level of the tube *AB*, taking care that no air is left in the flask. Place some aniline dye in the upper reservoir *AC* so as to colour the water. Heat the flask *BE* from underneath with a Bunsen burner. The water in the flask and the vertical tube *AB* becomes hotter than that in the reservoir and the longer bent tube. Hence the water rises from the flask along the tube *BA* and coloured water flows down the bent tube to take its place. A circulation of water round the tubes is thus set up and after a time the plain and coloured waters become mixed.

(c) Repeat Experiment (46) (a), using instead of the capsule of burning spirit a small vessel containing a freezing mixture. The index rises in the air thermometer, shewing that the bulb is cooled. This is due to convection. The water at the top is cooled and becoming denser sinks, thus reducing the temperature of the thermometer.

151. Hot water heating apparatus. The experiment 47 (b) of the last section illustrates the principle of a hot water heating apparatus. A pipe rises from the upper part of a boiler to a tank or reservoir at the top of the building which it is wished to heat. The downward pipe corresponding to *CDE* passes through a number of metal coils in the various rooms and finally enters the boiler again at the bottom. The water is cooled as it circulates through the pipes and its heat given up to the rooms.

This method of heating illustrates the three processes of conduction, convection and radiation. The heat passes from the furnace to the water by conduction through the plates; it is transferred to the interior of the pipes by convection, the hot water in its flow carries the heat; it passes to the exterior of the pipes by conduction and escapes into the room by radiation from the surface of the pipes and by convection in the currents of air which the warm pipes set up.

152. Convection in air. Light a piece of brown paper or blotting-paper, blow out the light and let the paper

smoulder. Hold a glass tube open at both ends over the paper; some of the smoke passes up the tube. Heat the upper part of the tube with a Bunsen flame or spirit lamp; the upward flow is very decidedly increased; a convection current is set up which draws the smoke up the tube.

Light a candle and place it at the bottom of a fairly wide lamp glass. The bottom of the glass may be covered with water as shewn in fig. 69 to prevent the ingress of air from below. The candle can obtain no supply of air and soon goes out. Cut a piece of card about the width of the glass, leaving a wider part at the top so that the card is T-shaped, and insert it at the top so as to divide the upper part of the tube into two portions. The candle will now burn, and, by holding a piece of smoking paper near, it will be seen that there is a draught down one half of the tube supplying fresh air and up the other half carrying off the products of combustion.

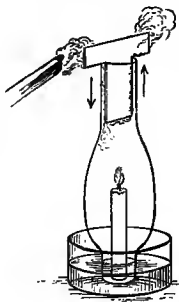


Fig. 69.

153. Ventilation. Convection currents produce an important effect on the ventilation of rooms and buildings. The hot air, heated by the fire, rises up the chimney and cold air enters under the door or round the window to take its place. The hot air in a room in which people are living rises to the top. It is for this reason that in a properly ventilated room the openings for the exit of foul air are at the top, while arrangements are made near the floor to allow the ingress of fresh air. In some cases the entering air is made to pass through a coil of pipes at the back of the fire-place to warm it and thus prevent the chilling effect which might otherwise be produced. In rooms heated by hot water the fresh air should pass over the water-pipes before it is admitted to the room.

154. The Trade Winds. Currents in the atmosphere are to a very considerable extent the result of convection. Thus the air near the equator gets heated and rises and the colder air from the north and south gets drawn towards the

equator to take its place. If the earth were at rest this would cause a steady north and south wind on the north and south side of the equator respectively. The earth however is turning round from west to east and a point on the equator is moving faster than one to the north or south of it. The air which is drawn towards the equator from northerly latitudes has a velocity towards the east which corresponds with the velocity of the point from which it was drawn, and this is less than the velocity of the points to which it is coming. It will appear therefore relatively to the earth to be moving from east to west, i.e. the northerly and southerly winds are thereby converted into north-easterly and south-easterly winds. These constitute the trade winds. There is a belt of calm just under the sun where the air is being drawn upwards, and this calm belt lies to the north or south of the equator according to the time of the year. The air which is thus drawn up passes away to the north and south as an upper current, descending again to the surface of the earth about our latitude in the northern hemisphere. This air coming from the equator has a greater velocity towards the east than the earth's surface in this latitude; it therefore appears to come from the south-west and constitutes the south-west wind which blows frequently. The direction and character of the winds at any place are of course much modified by local causes.

Ocean currents are to some extent also produced by convection, but the action of the wind on the surface is in this case the predominating cause.

EXAMPLES.

CONDUCTION AND CONVECTION.

1. A building is heated by hot water pipes; how does the heat get from the furnace of the boiler to a person in the building? What would be the effects on the temperature of the more distant parts of the building of coating the pipes near the boiler, (a) with woollen felt, (b) with dull black lead?

2. What is meant by the thermal conductivity of a substance?

An iron boiler 1.25 cm. thick contains water at atmospheric pressure. The heated surface is 2.5 sq. metres in area and the temperature of the under side is 120°C . If the thermal conductivity of iron is .2 and the latent heat of evaporation of water 536, find the mass of water evaporated per hour.

3. Two equal rods, the one of bismuth, the other of iron, are thinly coated with wax, and one end of each is raised to the same temperature. Describe and account for the phenomena observed in the two rods respectively.

4. Distinguish between the conduction, convection and radiation of heat, and describe experiments by which (a) the conductivity of two metals may be compared, and (b) the conductivity of one metal determined.

5. An iron boiler 1.3 cm. in thickness contains water at the atmospheric pressure. The heated surface is 3 sq. metres in area, and its under side is kept at a temperature of 115°C . Taking the thermal conductivity of iron as .2, find the quantity of heat entering the boiler in an hour.

6. An iron boiler $\frac{3}{8}$ inch in thickness exposes 60 square feet of surface to the furnace and 600 lbs. of steam at atmospheric pressure are produced per hour. The thermal conductivity of iron in inch lb. sec. units is .0012 and the latent heat of steam is 536. Find the temperature of the under side of the heating surface. Explain carefully why this is not the temperature of the furnace.

7. Define the thermal conductivity of a substance and describe some way of measuring it.

8. How many units of heat will be conducted in an hour through each square centimetre of an iron plate 0.02 cm. thick, its two sides being kept at the respective temperatures of 0°C . and 50°C ., the mean conductivity of iron being 0.12?

9. The opposite sides of a plate are kept at 0°C . and 100°C . by contact with ice and steam. Shew how to deduce the conductivity of the plate by observing the quantities melted and condensed. What are the practical defects of the method?

10. To the two sides of a metal vessel are soldered rods of bismuth and iron; to the rods a number of shot are attached by means of soft wax; the vessel is filled up with boiling water; state and explain what will occur.

11. The inside of the wall of a house is at 15°C ., and the outside at 0°C ., the wall is of stone, and 50 cm. thick. Find how much heat passes across it per square metre. The conductivity of the stone is .005 and the unit of heat is the quantity required to raise the temperature of one gramme of water one degree centigrade.

CHAPTER XII.

TRANSMISSION OF HEAT BY RADIATION.

155. Radiation. As has already been explained heat-energy is transmitted by radiation when it passes through a medium without raising its temperature. A medium which permits the passage of radiation is said to be diathermanous; in such a medium the energy does not exist as heat; one which will not permit of such passage is adiathermanous; radiant energy falling on an adiathermanous medium is transformed into heat; it is said to be absorbed by the medium and the temperature in consequence rises.

Radiant energy reaches us from the sun, being transmitted through the 92,000,000 miles between us and the sun in a period of some 16 minutes, but the space between us and the sun is not thereby warmed. This space is filled with a medium which is known as the ether: the sun has the power of producing motion in the ether, giving to the ether particles kinetic energy and setting up waves which travel outwards with great velocity, about 3×10^{10} cm. or 189,000 miles per second; any diathermanous substance can transmit these waves, though with a reduced velocity. An adiathermanous substance cannot transmit them. When they fall on such a substance the regular vibrations which constitute the waves are quenched, the energy of these vibrations passes into the energy of the irregular motions to which heat is due. Radiation is absorbed and a rise of temperature follows.

When radiation from the sun falls on us it excites two sensations, we feel heat and we see. The vibrations in the ether differ, among other points, in the rapidity with which they are executed. Vibrations of certain degrees of rapidity are capable of exciting the nerves of the eye, to these we give the name of Light. In order to produce the sensation of vision the ether particles must vibrate between 4×10^{14} and 7×10^{14} times per second. These same vibrations falling on other parts of the body excite motion in the nerves there. The energy of the motion is absorbed and heat is produced, but vibrations in which there are considerably less than 4×10^{14} oscillations in a second and which are too slow to affect the eye can produce the sensation of heat; to such vibrations the name of radiant heat is given; for their detection and measurement we use different apparatus to that which we employ in the case of light, and it is these vibrations in the main which we proceed to study experimentally in this chapter. In addition to the sensations of light and heat, radiation falling on a body can produce chemical changes, thus plants require a supply of radiant energy to enable them to assimilate the carbon in the air and grow.

Certain salts of silver are decomposed by the action of radiation and it is in consequence of this fact that photography becomes possible. These chemical changes may be produced by vibrations of the same period as those which affect the eye, i.e. by light. They can also be set up by the less rapid vibrations, or as is specially the case with the silver salts used in photography, they may be caused by vibrations which are too quick to disturb the optic nerve.

Thus radiant energy in the ether may become known to us in different ways, and different names, "thermal," "luminous," or "actinic," have been applied to it; these, however, are all names for the same thing manifesting itself to our sensations in different ways, according to the nature of the recipient on which it falls, and the rapidity with which the vibrations by which it is transmitted are executed. The second part of this book is concerned with the luminous effects of radiation; at present we are dealing with thermal effects, effects that is which shew themselves in a change in temperature of the

body receiving the radiation. The laws we are about to enunciate apply in the main equally well to Light and to invisible radiant energy. If however we are dealing with Light the effects are immediately visible, in the case of the invisible radiations we need some apparatus to render their presence sensible; if for example they fall on the bulb of a delicate thermometer the substance it contains expands and we see the motion of the index.

156. Means of measuring radiation. Various forms of apparatus can be used to shew the presence of invisible radiant energy. A sensitive thermometer will often be sufficient, or we may employ the differential air thermometer shewn in figure 10. In any case the bulb on which the radiation is to be received should be coated with lampblack; the reason for this will appear later. The differential ether thermoscope (fig. 70) may also be employed. This consists of two bulbs *A* and *B* like the cryophorus (fig. 56) connected by a glass tube and containing only ether and ether vapour. The one bulb *A* is coated with lampblack. When radiation falls on it, its temperature rises and the vapour pressure of the ether in the bulb *A* is increased; the consequence is that the ether rises in the tube leading to the bulb *B*: it may all be forced up into the bulb *B*, leaving only vapour in *A*, if the rise of temperature be sufficient.

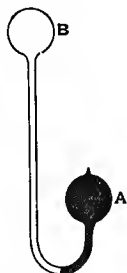


Fig. 70.

Another instrument which is used in radiation experiments is the thermopile.

If the ends of two pieces of wire of different materials, iron and copper say, be connected together, and then one of the junctions be heated, the other being left cold, an electric current is produced; this current can be easily measured by including in the circuit a galvanometer; if the difference of temperature between the junctions be not very great the current is approximately proportional to it. Now a very small current can be measured and hence a very small difference of temperature can be observed.

This is made use of in the thermopile; the metals used are antimony and bismuth, for the current produced by a small difference of temperature is greater for these metals than for others. A number of bars of the metals are arranged alternately as in fig. 71, *A* 1, 2 3, 4 5 being antimony bars, 1 2, 3 4, 5 *B* bismuth. The bars are soldered together at 1, 2, 3, 4, 5, and the ends *A*, *B* are connected to a galvanometer¹. If the junctions 1, 3, 5 be heated while 2 and 4 remain cool, a current passes from *A* to *B* through the galvanometer, while if the junctions 1, 3, 5 be cold while 2 and 4 are heated, the current through the galvanometer is from *B* to *A*. The current always passes from antimony to bismuth across the cold junction. The effect will be produced by a single junction, but by increasing the number the current is usually increased, though this depends on the galvanometer and the connecting wires. In the apparatus as usually made, a large number of junctions are connected up in square order as shewn in fig. 72, the contiguous bars of metal being electrically insulated from each other by strips of mica. In the figure are shewn the connecting screws to which the galvanometer wires are attached. The surfaces on which radiation is to fall are blackened.

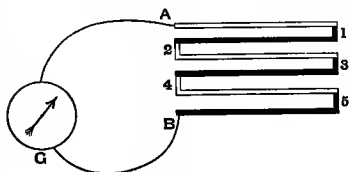


Fig. 71.

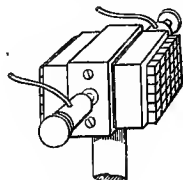


Fig. 72.

For some experiments the ends of the thermopile can be fitted with hollow cones. The interiors of these cones may be bright and polished; in this case all the rays which enter the open ends of the cones are reflected from their polished interiors to the face of the thermopile, and since the area

¹ A galvanometer is an instrument in which an electric current causes a small delicately suspended magnet to move (see Books on Electricity), and the current can be measured by observing the deflection of the magnet. In Thomson's Mirror Galvanometer a small mirror is attached to the magnet and a spot of light is reflected by the mirror on to a scale. The motion of the magnet is indicated by the motion of this spot.

of the open end of the cone may be considerably greater than that of the face, a much larger amount of radiation is concentrated on the face than otherwise would reach it: in other cases the interior of the cones is covered with a dull black surface; a piece of dead black paper cut into the form of a sector of a circle and fitted inside will form such a surface. In this case the rays which fall on the black surface are absorbed by it. The only rays which can reach the thermopile are those which come from that portion of space which would be visible to an eye situated where the face of the thermopile is and looking out through the cone. There are various other forms of apparatus for measuring radiation. Prof. Boys' radiomicrometer is a very sensitive thermopile and galvanometer combined. It will readily indicate the radiant energy received by it from a candle placed at the far side of a large hall.

In experiments on radiation various sources of radiant energy may be used. For some purposes a Bunsen lamp burning with its usual non-luminous flame will serve. A piece of copper or platinum placed in such a flame becomes red hot and may be used for the source. For other purposes a ball of metal heated red hot in a fire will serve, while for others again a Melloni cube, which is merely a tin or brass box filled with boiling water, is convenient. In the last case it is often desirable to have some means to keep the water boiling, and at the same time to protect the thermopile or thermometer from direct radiation from this source. Screens can generally be arranged to secure this result.

With this apparatus various experiments may be made on radiation.

157. Transmission of Radiant Energy.

EXPERIMENT (48). *To shew that in a uniform medium radiant energy travels in straight lines.*

(a) Take any small source of radiation, a Bunsen burner with a piece of platinum for example, and place a small screen¹

¹ Such a screen may be made by fastening two pieces of tin sheet so that they can stand in a vertical position with their planes parallel and separated by an air-space 1 to 2 cm. in breadth.

opaque to radiation at a distance of 25 or 30 cm. from the source.

Place the thermopile or the blackened bulb of the thermoscope at say 25 cm. behind the screen and within the shadow space formed by drawing lines from the source to points on the edge of the screen. No effect is noticed. A straight line from the source to the thermopile will cut the screen and radiation travelling along such a line is stopped by the screen. Now move the thermopile so that it is just outside this shadow, i.e. so that a line can be drawn from it to the source without cutting the screen. Its temperature immediately rises, radiation travelling in a straight line can now reach the thermopile.

(b) Place a screen with a small hole in it fairly near to the source; place a second screen also with a small hole (say 1 to 2 cm.) in diameter at a little distance away. On placing the thermoscope so that a line can be drawn from it through the two holes to the source, it is affected; in other positions no rise of temperature is indicated.

(c) Place the first screen near the source and in front of it place a metal or card-board tube some 4 or 5 cm. in diameter and some 40 or 50 cm. long in such a position that its axis is directed through the hole towards the source. The inside of the tube should be blackened. The only radiation which can reach the thermopile is that which can travel directly down the tube, and in consequence the thermopile is only affected when directly opposite the end of the tube.

158. Diathermanous and Adiathermanous substances. The percentage of radiant energy, incident on a plate of any material of given thickness, which is transmitted by that plate, is different for different materials and depends also on the nature of the radiations themselves. Thus glass which is transparent to the luminous vibrations is nearly opaque to the invisible vibrations of longer period; the same is true of a solution of alum in water. Rock salt on the other hand allows the passage of radiation of every period to about the same extent, while a solution of iodine in bisulphide

of carbon which is opaque to luminous radiation transmits the invisible rays of longer period very freely.

In experiments on radiation we must remember that the visible effect depends on waves whose periods lie within certain definite limits, while the thermopile or thermoscope, covered with lampblack, measures the total quantity of radiant energy which falls on it.

EXPERIMENT (48 A). To compare the relative diathermancy of plates of various materials.

Place the thermopile at a convenient distance—say some 50 cm.—from a source of radiant energy. Interpose between the two a screen with a hole in it, which can be covered by the plate whose diathermancy is required, and a stand on which this plate can be supported so as to cover the hole. The numerical results obtained will depend on the nature of the source employed: suppose it to be a spiral of platinum wire or a piece of platinum foil made incandescent by a Bunsen burner or spirit lamp. Have ready a number of plates of the different materials to be examined which can be placed on the stand so as to cover the hole; the plates should all be of the same thickness. Allow the radiation from the source to fall freely on the thermopile, and observe the deflection of the galvanometer when it has become steady. Interpose a plate of some material, say rock salt. The deflection is somewhat reduced; the ratio of the two deflections gives the ratio of the total energy transmitted by the rock salt plate to the total energy falling on it. According to Melloni for a plate .25 cm. in thickness this is about 92 per cent. Of the remaining 8 per cent. a small part is absorbed by the rock salt, the rest is reflected back to the source from its surfaces. Perform the same experiment for the various other substances; the proportion transmitted varies very greatly, the percentage reflected does not differ greatly for the various plates, hence the percentage absorbed is very different. Thus using as the source the glowing platinum, a plate of glass, which permits the passage of nearly all the luminous vibrations, transmits only about 28 per cent. of the total radiation, while a solution of alum which is transparent to light allows only about 2 per

cent. of the whole energy to pass, and this compared with 92 per cent. transmitted by the rock salt. If the source be changed to one which emits a smaller proportion of luminous vibrations, say a sheet of copper heated to 400° , the results are more striking still. The rock salt still transmits 92 per cent. while only 6 per cent. can now pass through the glass; the amount transmitted through the alum is too small to be measured.

Thus in this case nearly all the radiant energy is non-luminous and nearly all can pass the rock salt.

The following experiment illustrates the distinction between the visible and invisible radiations very forcibly. Bisulphide of carbon is fairly transparent to all radiation; by putting a little iodine in the bisulphide it becomes quite opaque to light but still transmits a very large percentage of the total radiation. Thus Tyndall placed a cell containing bisulphide of carbon between an electric lamp and a thermopile, and observed the current produced. He then replaced this cell by a similar one containing the iodine solution, and found in this case that the energy transmitted was $\frac{9}{10}$ ths of the previous amount. According then to this observation only $\frac{1}{10}$ th of the total energy emitted by the lamp is capable of producing vision, the remaining $\frac{9}{10}$ ths cannot affect the eye as light¹.

It has been shewn also by Tyndall that the diathermancy of different gases for radiation from a source such as the glowing platinum spiral is very varied. Thus the absorption of carbonic oxide is about 750 times as great as that of air or oxygen, while that of olefiant gas is more than 10 times as great again, or about 8000 times that of air.

The following table gives some of Melloni's results for different materials and different sources; the numbers given

¹ In this and other experiments to be described shortly with the iodine solution, the sides of the cell used to hold the liquid through which the radiant energy has to pass should be of rock salt. If they are of glass, a large proportion of the radiation will be absorbed by the glass.

indicate the total percentage transmission; the plate in each case being .25 cm. or $\frac{1}{10}$ th inch thick.

	Oil Lamp	Incandescent Platinum	Copper at 400°	Copper at 100°
Rock salt	92.3	92.3	92.3	92.3
Fluor spar	72	69	42	33
Iceland spar	39	28	6	0
Glass	39	24	6	0
Quartz	38	28	6	0
Alum	9	2	0	0
Ice	6	0	0	0

Since as measured photometrically nearly all the luminous energy passes the alum, we infer that of the total energy emitted by the Locatelli oil lamp only 9 per cent. is luminous.

These numbers only hold for the thickness given; the amount transmitted decreases rapidly as the thickness increases. We notice also that the percentage transmitted depends on the nature of the source, all the substances except the rock salt exercise a selection in the vibrations they transmit.

159. Reflection of Radiant Energy.

EXPERIMENT (49). *To shew that radiant energy can be reflected.*

(a) Arrange some reflecting surface, a sheet of glass or tin or a thin piece of wood with its plane vertical, in such a way that it can easily be turned about a vertical axis through its centre¹. Attach a light lath or pointer with its length normal, or perpendicular, to the reflecting surface, in such a way that it moves over the table, tracing out a circle as the reflector is turned round. Adjust two tubes such as that described in Experiment 47 (c) so that their axes may be horizontal and may meet about the middle of the reflecting plate, supporting them in suitable stands, and place a source of

¹ This can be done by drilling a hole with a centre-bit into a block which can be clamped on to the table, and fitting a round peg into this hole. The reflecting surface can then be secured to this peg.

radiation, the hot ball suppose, at the end of one tube, the thermopile at the end of the other shading the latter from direct radiation. For most positions of the reflector the effect on the thermopile will be very small, but on turning it round, a position can be found in which the galvanometer needle is considerably deflected; when the deflection is greatest, it will be found that the pointer just bisects the angle between the two tubes. The radiant energy travelling in straight lines down one tube is reflected along the other, and the angle between the normal to the reflecting surface and the incident radiation is equal to that between the normal and the reflected radiation. The experiment can be varied by altering the mutual inclination of the tubes; the pointer always bisects the angle. Fig. 73 shews the arrangement of the apparatus.

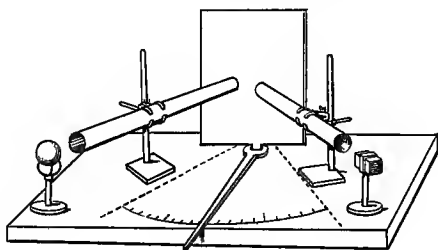


Fig. 73.

(b) Take a source of radiation, such as the ball after it has become non-luminous, but while it is still hot, and place the thermopile at some distance—say a metre—away. The effect will probably be small. Take a long metal tube such as has been already used, but polished brightly inside, and place it between the two, one end of the tube being close to the ball: the galvanometer deflection or the indication of the thermoscope becomes much more marked. A large part of the radiation diverging from the ball enters the tube, and after one

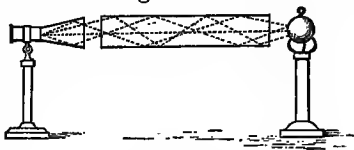


Fig. 74.

or more reflections from the sides as shewn in fig. 74, falls on the thermopile and raises its temperature.

(c) The laws of reflection can be more easily verified in the case of light: we can shew by the following experiment that the invisible radiation follows the same path as the visible.

Take two large concave mirrors¹. Place a source of light, a small gas flame, at the principal focus of one mirror. Allow the parallel beam which proceeds from the mirror to fall on the second mirror placed as far away as is convenient: the radiation is reflected to the principal focus of this mirror, and on placing at this focus a small piece of paper, an image of the source is formed on the paper. Replace the source of light by the heated ball, and bring the blackened bulb of the thermoscope or the face of the thermopile near the second mirror. No very large effect will be produced, except when the face of the thermopile is in the position which in the previous experiment was occupied by the sheet of paper. The rays from the ball fall on the first mirror and are there reflected in a parallel beam; after reflection from the second mirror they converge on the thermopile to the same spot as the luminous rays converged when producing the luminous image.

The arrangement is shewn in fig. 75.

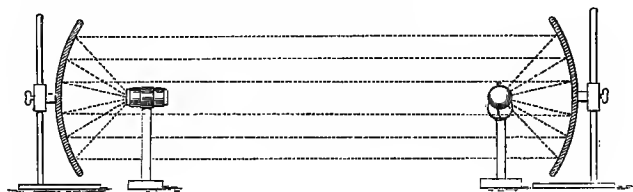


Fig. 75.

¹ These are usually made of copper beaten into a spherical form, polished on the inside and coated with silver, and should be some 50 cm. across. A point midway between the centre of the surface and the centre of the sphere is called the principal focus. If a luminous source be placed at this point the rays diverging from it are reflected from the mirror and proceed in a parallel beam parallel to the line joining the centre of the sphere and the principal focus. See *Light* § 50.

If in the above experiment the hot ball be replaced by a piece of ice, or a small beaker or other vessel containing a freezing mixture so as to be at a low temperature, the thermoscope in the focus of the second mirror will shew that the temperature then falls. We shall have to discuss later some of the theoretical consequences of this.

In any of the above experiments a luminous source such as an electric lamp may be employed and the luminous rays cut off by a solution of iodine in bisulphide of carbon.

160. Refraction of Radiant Energy. Light falling obliquely on the surface of glass, water, or any other transparent medium is refracted or bent out of its course according to certain definite laws (see *Light* § 32). The same is true for invisible radiation. Again the amount of refraction depends on the nature of the light, the very rapid vibrations are most bent.

EXPERIMENT (50). *To shew that radiant energy can be refracted.*

(a) Allow the rays of light from a bright source to pass through a narrow slit and to fall on a screen, thus producing a white patch on the screen. Interpose in the path of these rays a prism of some transparent material¹. A coloured patch is now visible on the screen some distance to one side of the original white light. This patch is called a spectrum. It will be found that the bending or deviation of the light has taken place towards the thick end, and away from the edge of the prism, and that the patch is coloured red on the side nearest the original white light, blue or violet on the other side. Now take a linear thermopile, one, that is, which has a single row of junctions arranged in a line one above the other. Since we know that glass absorbs a large fraction of the incident radiation, to get a large effect we use a rock salt prism. Place the thermopile in the spectrum, its length being parallel to the edge of the prism, and move it through the spectrum from the violet toward the red. The galvanometer is deflected, the deflection increasing as the red end is ap-

¹ See *Light* 43.

proached. Continue to move the thermopile beyond the visible red of the spectrum. The needle is still deflected, shewing that the invisible radiation, like the visible, is refracted by the prism but to a less extent. The visible radiation may be cut off by the iodine solution; if the thermopile be left in the invisible part of the spectrum, a deflection is observed of nearly the same extent as before, while if it be placed where the visible part previously was, the effect is much reduced.

(b) It is owing to refraction that an image of a luminous source can be produced by a lens. The simplest observation with a burning glass suffices to shew that the radiant energy from the sun can be concentrated by a lens to the same focus as the light; that the effect is not due to the luminous vibrations can be proved by quenching them with the iodine solution contained in a rock salt cell. Thus if a thermopile or thermoscope be placed at some distance from a source, the effect will be very small, but if a lens of rock salt be inserted so as to focus an image of the source on the pile, a very large effect is produced, and that, even though the luminous energy has been removed by the iodine solution. Hence radiant energy can be both reflected and refracted.

161. Intensity of Radiation at a point. The heating effect produced over a given surface by radiation will be proportional to the quantity of radiant energy which falls on it. It may often happen that this incident radiation is uniformly distributed over the surface, so that if we suppose the surface divided up into any number of equal small portions, the same amount of energy is received by each portion. The total energy received will then be found by multiplying the amount which falls on each small area by the number of such areas in the surface. If we suppose each of the small areas to be one square centimetre, the number of such areas will be the number of square centimetres, and the total amount of radiation received will be obtained by multiplying together the amount received by each square centimetre and the number of square centimetres; or stating it differently, the amount of radiation falling on each square centimetre is

obtained by dividing the total amount of energy falling on the surface by the area of the surface in square centimetres.

The amount of radiation received by a surface of given area will depend on the angle between the rays and the surface. More energy will be received by a square centimetre of surface, when it is placed at right angles to the direction in which the rays are travelling, than will reach it when placed obliquely to those rays.

Definition of Intensity of Radiation. *The amount of radiant energy, which falls on each square centimetre of a surface, placed normal to the rays and receiving radiation uniformly distributed, is called the intensity of the radiation at each point of that surface.*

If then we know the intensity of the radiation at each point of a given surface, we find the whole amount of radiation falling on it by multiplying the intensity by the area of the surface; if we know the total amount falling on the surface, we find the intensity, the distribution being uniform, by dividing the total amount by the area of the surface in square centimetres¹.

162. Radiating power of a source of radiant energy. The amount of energy radiated by two different sources in the same time may be very different. Much more energy is emitted per second from a white hot metal ball than from the same ball when nearly cold. Let us take a case in which the radiation is uniformly distributed all round the source. Then if one source is emitting in a given time twice as much energy as a second, twice as much energy will fall in that time on a given area—1 square centimetre say—placed at a given distance from the first source as falls on another square centimetre placed similarly at the same distance from the second source. The amount of energy received by an area in a given position is proportional to the total amount of

¹ If the radiation be not uniformly distributed, the quotient just found will give the average intensity over the surface: the intensity at each point of the surface will vary from point to point, and may be calculated by finding the amount of energy falling on any very small area taken so as to include the point and dividing that amount by the area.

energy radiated from the source. The radiating power of a source may therefore be properly measured by the amount of radiation falling on a definite area placed in a definite position with regard to the source.

Definition of the radiating power of a source.

The radiating power of a source is measured by the amount of radiation which falls on an area of 1 square centimetre placed normal to the rays at a distance of 1 centimetre from the source.

When then we say that the radiating power of a source is I we mean that I units of radiant energy fall on an area of 1 square centimetre placed normal to the rays at a distance of 1 centimetre from the source.

163. Rectilinear propagation. Law of the inverse square. If we suppose radiation to travel outwards from a source in straight lines, it is clear that a less amount of energy will fall on a given area at a distance from the source than is incident on the same area when moved closer to the source. We can investigate the law of this diminution on the assumption that the energy travels outwards in straight lines.

For let $ABCD$, fig. 76, be a square aperture each edge of which is 1 cm. placed at some little distance from a small source of radiant energy O , so small that we may treat it as a point.

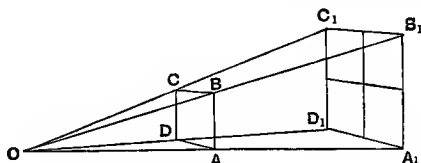


Fig. 76.

Place a second screen behind the first and let the lines OA, OB, OC, OD produced cut this second screen in $A_1B_1C_1D_1$, respectively. Then the amount of radiation which would fall on $ABCD$ if there were no hole is intercepted by

$A_1B_1C_1D_1$, it is thus distributed over a greater area and the intensity at each point of that area is therefore proportionately less. Now if OA_1 is twice OA , so that the second screen is twice as far from the source as the first, each side of the second square is clearly twice as great as the sides of the first, thus the area of the second square is 2×2 or 2^2 times that of the first. It is therefore 2^2 square centimetres, and the amount of radiation falling on each square centimetre is $1/2^2$ of that which falls, per square cm., on the first. Now suppose the screen moved further away until it is three times as far from O as $ABCD$. Each side of the square formed by producing OA , OB , etc. will now be 3 cm.; the area on which radiation falls still assuming the rectilinear propagation will be 3^2 square cm., and the intensity of radiation will now be $1/3^2$ of its original value; by doubling the distance the intensity of the radiation is reduced in the ratio of 1 to 2^2 , by trebling it the reduction is as 1 to 3^2 .

Thus the intensity of the radiation at a point due to a given source is inversely proportional to the square of the distance of the point from the source.

This is known as the law of the inverse square for radiant energy and has been deduced from the fact of the rectilinear propagation: we will proceed to give some direct experimental illustrations of its truth.

The following mathematical expression of the law however may be useful. Let I be the radiating power of the source, i.e. the amount of radiation which falls normally on 1 sq. cm. placed at a distance of 1 cm. Thus I the radiating power of a source is also the intensity of radiation at a point 1 cm. distant from the source. Now if the aperture $ABCD$ in fig. 76 above be 1 cm. distant from O the total amount of radiation which passes the aperture is I , and, if the screen be r cm. away from O , each side of the square on which this radiation falls is r cm. The amount of radiation I therefore falls on an area of r^2 sq. cm.; the amount falling on unit area is therefore I/r^2 , and this amount measures the intensity of the radiation at each point of that area. Thus the intensity of radiation at a point r cm. distant from a source is found by dividing the radiating power of the source by the square of the distance between the point and the source. Thus we have the formula: Intensity of radiation at a distance r from a source of radiating power I is equal to $\frac{I}{r^2}$.

164. Experimental evidence for the law of the inverse square.

EXPERIMENT (51). *To prove that the intensity of the radiation at a point, due to a given source, is inversely proportional to the square of the distance of the point from the source.*

(a) Take a small source of radiation, the platinum spiral heated in the Bunsen burner or the central bright spot of an oxyhydrogen lime light. Place a screen with a small hole in front so as to cut off the radiation from all but a small area of the source; allow this to fall on a thermopile placed at, say, 100 cm. distance from the source. Observe the current through the galvanometer. This is approximately proportional to the rise of temperature of the pile and this again is proportional to the radiant energy falling on it. Thus the energy falling on a given area, the face of the thermopile, can be measured. Let us call the distance d and the current i . Alter the distance to say 80 cm. and again observe the current. Write down in columns the corresponding values of the distance, the current and the squares of the distance. Then form the series of products obtained by multiplying together the corresponding values of the current and of the square of the distance. It will be found that these products are nearly the same, thus the intensity of radiation, measured by the current, multiplied by the square of the distance, is constant, i.e. the intensity is inversely proportional to the square of the distance.

The following table gives Melloni's values, and illustrates how to enter the results.

d	i	d^2	$i \times d^2$
100	10.34	10000	10340
70	21.10	4900	10339
60	28.70	3600	10343

Thus the law is clearly verified.

(b) For this experiment a large radiating surface is required. A tin vessel (fig. 77), whose dimensions are about $75 \times 75 \times 10$ centimetres, may be used. One face is covered with lampblack and the whole filled with boiling water. The

hollow cone is fitted to the thermopile and lined with the black paper, the instrument is placed near the heated surface, as at P . If the cone be produced to meet the surface it will cut it in the circle AB . Radiation from points within this circle alone can fall on the pile. Radiation from points outside the circle falls on the blackened paper but is absorbed by it and does not reach the thermopile. Observe the deflection produced.

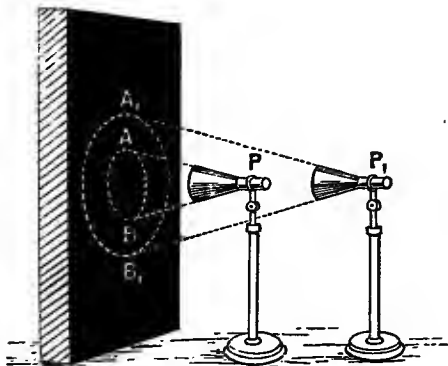


Fig. 77.

Move the thermopile back until it is in a position such as P_1 and observe the deflection; it will be found to be the same as previously, and to remain unaltered whatever be the distance to which the thermopile is withdrawn, provided only that the whole of the circle such as A_1B_1 lies on the blackened surface.

As the thermopile is withdrawn the intensity of the radiation received by it from each point of the source decreases, but the area from which the radiation is received increases. The total amount received depends on the product of these two factors, and this remains constant. Thus the intensity of radiation received from any one point is inversely proportional to the area of the circle, and this area is directly proportional to the square of the distance between P and the surface. Hence the intensity of the radiation received by the thermo-

pile from any one point of the surface is inversely proportional to the square of the distance between that point and the thermopile.

165. The Emission of radiation. We have defined the radiating power of a source as the total amount of radiant energy it emits per second. It is difficult to measure this absolutely as so many ergs or units of work, it is much more easy to compare the amounts of radiation emitted by two or more surfaces and to shew that different surfaces although at the same temperature emit different amounts of radiation.

EXPERIMENT (52). *To compare the amounts of radiant energy emitted by different surfaces.*

(a) Take two small tin canisters with lids to them. Bore two holes in each lid through which a thermometer and stirrer may be inserted. Coat the outside of one with lampblack and leave the second bright. Support each canister on three small cork feet like those employed for the Calorimeter, fig. 15. Fill each with the same mass of hot water at the same temperature, say 60° or 70° , and replace the lids. Take the temperature with a thermometer at intervals of say 5 minutes, stirring the water each time to secure uniformity. It will be found that the blackened canister loses heat much more rapidly than the one with a bright surface.

(b) Place a Leslie's cube on a stand which can turn round a vertical axis as shewn in fig. 78 and place the thermopile with the polished cone near to the cube. One of the faces of the cube—a metal box about 10 cm. in side with a lid—may be coated with lampblack, another painted white or varnished, a third roughened and the fourth polished. Fill the cube with boiling water, replace the lid and turn the polished face to the thermopile, very little effect is produced; note the deflection. Turn the cube so that the radiation from the rough face is received; the deflection is considerably increased. The varnished face produces a greater effect still and the lampblack face the greatest of all. Thus these different faces, though at the same temperature, emit very different amounts of radiation, and these radiating powers can be compared by comparing the respective deflections.

These last two experiments have shewn us that lampblack radiates out more heat at a given temperature than any of the other surfaces with which we have worked. It is convenient therefore to take the radiation of a lampblack surface as a standard with which to compare the radiation from any other

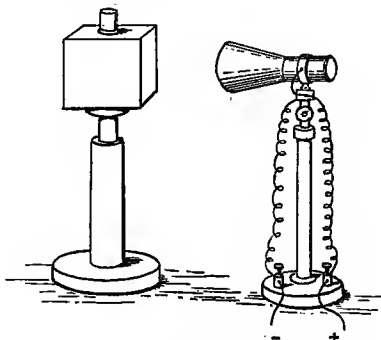


Fig. 78.

surface. The radiation emitted by any surface will be less than that emitted by a lampblack surface at the same temperature; the ratio of the two amounts may be spoken of as the emissive power of the surface.

Definition. *The emissive power of a surface is the ratio of the amount of radiation emitted in a given time by the surface to the amount emitted in the same time by an equal area of a lampblack surface at the same temperature¹.*

166. The absorption of radiation. We have seen that different surfaces at the same temperature emit different amounts of radiation. We shall now shew that they absorb different proportions of the radiation which falls on them.

¹ In the above we have drawn a distinction between the radiating power of a surface, i.e. the actual amount of energy it is emitting per second, and the emissive power, the ratio of the amount emitted by the surface to the amount emitted by lampblack. This distinction is not always observed.

If the substance be partly diathermanous some of the incident energy is absorbed by it, raising its temperature, some is transmitted; if it be adiathermanous the whole incident radiation is absorbed. In what follows, we suppose that the substance is practically adiathermanous, that the amount of energy which can traverse it is negligibly small. The results will to some extent depend on the thickness of the layer of substance, which is emitting or absorbing the radiation.

Definition. *The absorbing power of a surface is measured by the ratio of the amount of radiation absorbed by the surface in a given time to the whole radiation which falls on it in that time.*

A surface which absorbs the whole of the incident radiation is said to be perfectly black. Lampblack is very approximately a perfectly black surface, thus the absorbing power of lampblack may be taken as unity. We may give therefore a second definition of absorbing power identical with the above, but in form resembling more closely the definition of emissive power.

The absorbing power of a surface is the ratio of the amount of radiation absorbed by the surface in a given time to the amount which would be absorbed in the same time by an equal area of a lampblack surface under the same conditions.

EXPERIMENT (53). *To prove that different surfaces have different absorbing powers, and to compare their values.*

(a) Take two air thermometers such as that shewn in fig. 8. Coat the bulb of one with lampblack, that of the other with tinfoil, or better still, with silver. Expose both equally to radiation, e.g. by placing them close together in front of a fire or stove. The temperature of the lampblack instrument rises considerably, that of the silvered instrument is but slightly affected.

(b) Repeat the experiment with the differential air thermometer fig. 10, coating one bulb with lampblack, the other with silver: a similar result is noticed.

(c) In fig. 79, *AB*, *CD* are two sheets of tin on suitable stands, one coated with lampblack, the other bright. To the centre of the back of each sheet, a piece of bismuth is soldered

as at *K*, and wires are led away to a galvanometer at *G*. The plates are connected by a wire at the top and are placed opposite to each other at some 50 or 60 cm. apart. Between the plates a source of radiation such as the hot copper ball, or a gas jet, is placed. A thermo-electric junction is formed by the contact of the bismuth and the tinned iron, and if the junctions on the two sheets be unequally heated a current is produced, and the galvanometer needle deflected. If the ball be midway between the sheets, the temperature of the blackened sheet is raised very considerably above that of the other; in

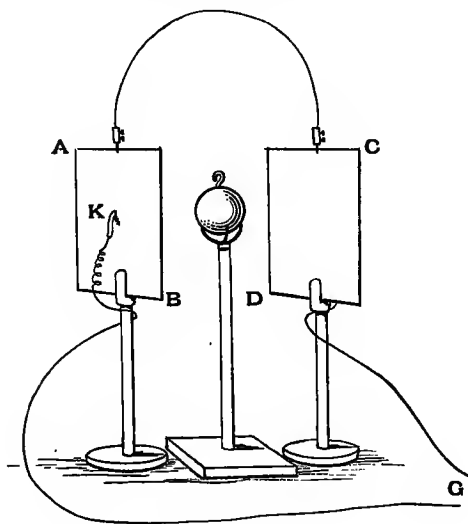


Fig. 79.

order that no current may be produced the source must be very considerably nearer the bright sheet. The blackened surface absorbs a much greater proportion of the incident radiation than the bright, and by measuring the distances of the ball from the surfaces when there is no current, squaring these and taking their ratio, a rough estimate of the absorbing power can be obtained.

(d) More accurate measures of absorbing powers are obtained by coating the thermopile itself with very thin layers of the various substances which are to be examined.

By comparing the results of the experiments described in the last two sections, it is found that good absorbers also emit radiation well. Thus lampblack which absorbs all the incident radiation was found to emit more than any other surface; bright surfaces which emit very little, absorb very little, and in fact, when accurate observations are made it is found that the absorbing and emitting powers of any surface at a given temperature are the same: if a surface emits 20 per cent. of the energy emitted by lampblack it will absorb 20 per cent. of the energy which would under the same circumstances be absorbed by lampblack, i.e. 20 per cent. of the whole incident radiation.

Tyndall made a number of experiments by coating the faces of a Leslie cube with powders of different materials, and obtained numbers which lead to the following results.

Substance.	Absorbing power.	Emissive power.
Rock salt	·319	·307
Fluor spar	·577	·589
Red oxide of Lead	·741	·707
Oxide of Cobalt	·732	·752
Sulphate of Iron	·824	·808

The numbers in the two columns are not greatly different.

The following experiment, a modification of one of Ritchie's, will shew the equality between the absorbing and emissive powers of a surface.

EXPERIMENT (54). *To shew that the absorbing and emissive powers of a surface are equal.*

Take two plates arranged as in fig. 79. Coat one with lampblack and polish the other. Take a Leslie cube of the same material as the plates and treat two opposite faces in the same way as the plates, polish one and blacken the other. Place the cube between the plates, turning the polished face

of the cube to the blackened plate and *vice versa*, and fill the cube with hot water. It will be found that there is no effect on the galvanometer when the cube is midway between the plates.

The blackened face radiates out more than the polished face; but the radiation from the blackened face falls on the polished plate which absorbs only a small fraction of it, while the radiation from the polished face falls on the blackened plate which absorbs nearly all. The total amount of radiation absorbed is the same in the two cases since the temperatures of the two junctions are unchanged, and this can only be the case if this radiation from the polished face bears to the radiation from the blackened face the same ratio as the absorption of the polished face bears to that of the blackened face.

Ritchie in his experiments used, instead of the two plates, two metal vessels, fig. 80, with flat faces. These constitute the bulbs of a differential air thermometer. The Leslie "cube" takes the form of the cylindrical box with flat ends, and is mounted on a stand, which as shewn in the

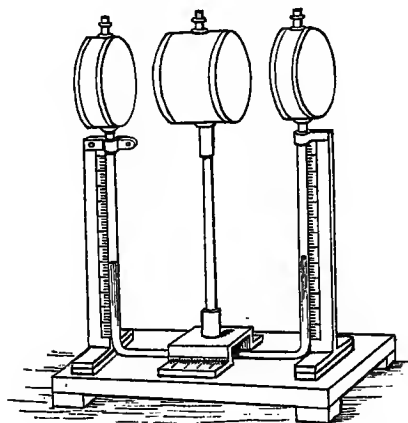


Fig. 80.

figure is fitted with a scale to determine its position between the bulbs.

167. The Reflection of Radiation. We have seen that radiant energy is reflected from a surface on which it is incident. The proportion however which the amount of radiation reflected bears to the incident radiation is different for different surfaces and depends also on the angle of incidence. Moreover, when radiation falls on a surface, that surface becomes heated, and in consequence emits radiation itself; thus the total radiant energy incident on a thermopile placed to receive reflected radiation is made up of the radiation regularly reflected, and the radiation emitted by the surface and diffused in all directions.

Definition. *The reflecting power of a surface is the ratio of the amount of radiation regularly reflected by the surface to the amount of radiation incident on the surface when the incidence is direct¹.*

When radiant energy falls on a surface, part is reflected, and part is refracted into the surface. If the surface is adiathermanous, this refracted portion is absorbed; if it is diathermanous the refracted portion passes through. With most substances the effect is intermediate between these two.

Now for adiathermanous substances the absorbing power is the ratio of the amount entering the surface to the amount incident, the reflecting power is the ratio of the amount reflected to the amount incident, but the amount entering and the amount reflected together make up the whole incident radiation. Hence the sum of the absorbing power and the reflecting power is equal to unity.

Thus we see that substances which absorb radiation readily reflect little and *vice versa*.

Observation bears out this result; the bright tin surface which absorbs very little radiation is a good reflector, the dull black surface which absorbs nearly all is a very bad reflector.

¹ By direct incidence it is meant that the incident rays are at right angles to the surface.

168. Absorption, Emission and Reflection of Radiation. Accurate experiments shew also that the result just stated is true, numerical measurements of absorbing power have been made and accord with the statement. Now the absorbing power of a perfectly black body is unity. We may therefore state the law thus:

Absorbing power of an adiathermanous substance + reflecting power of the same substance = absorbing power of a lampblack substance ; or again, since the absorbing power and the emissive power are the same,

Emissive power of a surface + reflecting power of the same surface = emissive power of a perfectly black surface.

If the substance examined be partly diathermanous, so that an appreciable portion of the incident radiation can pass through, we shall have also to consider the transmitting power of the surface.

Definition. *The transmitting power of a plate is the ratio of the amount of radiation which passes through the plate to the amount which is incident on it.*

The transmitting power was measured in Section 158, Experiment 48 A. It depends on the material and on the thickness of the plate. For such diathermanous substances we have to add the transmitting power to the left-hand side of the equations just given and write emissive power + reflecting power + transmitting power = emissive power of a black body. Exact experiments prove that these laws hold not only for the total radiation emitted and absorbed but for each kind of radiation. Thus some substances exercise a selective absorption, they absorb vibrations of certain periods and transmit those of other periods. These same substances emit readily vibrations of the first set of periods but not those of the other set.

169. Observations on Absorption, Emission and Reflection. The relation between absorption, emission and reflection can be illustrated by many examples. Thus if a piece of china with a dark pattern on a light ground be heated in a clear fire the dark pattern stands out bright on a

less bright ground. Or again make a cross or mark with ink on a piece of platinum foil or other bright surface and heat the whole over a Bunsen flame; the mark appears much brighter than the rest. Dark clothes are hotter than light, for the dark surface absorbs the radiation, the light surface reflects a large portion of it. The action of a glass fire-screen illustrates the selective absorption of glass, the luminous vibrations are transmitted but these contain only a small portion of the energy radiated from the fire. The same action is shewn in a greenhouse. The luminous vibrations pass through the glass, they are absorbed by the plants and objects inside, much of the energy they convey is radiated out again in invisible radiation of long period; to these the glass is opaque, and the energy is thus kept inside the hothouse and raises its temperature considerably.

Aqueous vapour is opaque also to vibrations of long period; hence it is that a damp cloudy night is apt to be warm, the free radiation of energy from the earth into space is prevented. Clear nights in winter or early spring are often frosty because radiation out into space can continue sufficiently to cool the earth down to the freezing-point.

Again a teapot or pewter hot-water jug is kept bright. Loss of heat by radiation is thus prevented; for the same reason the front part of a copper kettle is polished, the back towards the fire being left dull. The reading of a black bulb thermometer placed in the sun is much higher than that of an ordinary clear glass thermometer lying beside it, whilst on a clear night the black bulb instrument falls below the other.

170. Law of cooling. Common observation tells us that the rate at which a body loses heat by radiation depends upon its temperature and the temperature of the surrounding space. A red-hot ball cools more rapidly through 10° or 20° than the same ball when at a temperature of 40° C. or 50° C. The rate of cooling also depends on the surface. Some experiments led Newton to believe that the quantity of heat emitted per second by a body was proportional to the difference in temperature between the body and the surrounding space. So that if the space be at 15° C. a body at 35° C.— 20° C.

above the space—would radiate out twice as much heat in a second as the same body when at 25° or 10° C. above the space. This result is known as Newton's law of cooling—more exact experiment has shewn that it is only true for small differences in temperature between the hot body and the space.

EXPERIMENT (55). *To examine the law of cooling of a hot body.*

The hot body may conveniently be a small flat rectangular copper vessel about $7 \times 7 \times 1$ centimetres fitted with a lid through which pass a thermometer and a stirrer.

The vessel is coated with lampblack and filled with hot water. It is then suspended by silk threads or india-rubber bands inside a larger copper vessel blackened on the inside. This vessel can if desired be placed in a large tub of water and its temperature will thereby be maintained nearly constant; for many experiments it will however remain at a sufficiently constant temperature if exposed to the air of the room.

Keep the water in the inner vessel gently stirred and read the thermometer as the temperature falls from about 70° at intervals of, say, 1 minute; after a time, when the rate of fall has become slower, the intervals may be considerably longer.

Plot the results as a curve representing the times by horizontal lines parallel to OA and the observed temperatures by vertical lines parallel to OB . The curve will have the form shewn in fig. 81.

Let P_1, P_2 be two points on the curve corresponding to the temperatures P_1N_1, P_2N_2 at times ON_1 and ON_2 .

Draw P_2R horizontally. The amount of heat lost in the

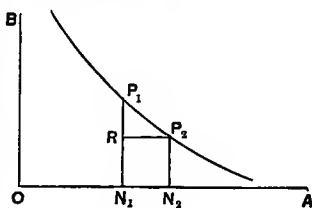


Fig. 81.

interval N_1N_2 is proportional to the fall of temperature, this is measured by P_1R . The average rate at which heat is being lost then, during this time, is proportional to the ratio of the fall in temperature P_1R to the time RP_2 or N_1N_2 , and thus can be found from the diagram. If Newton's law held, this rate of fall should always be proportional to the difference between the temperature of the hot body and that of the enclosure; this is not found to be accurately the case.

171. Rate of loss of Heat, Coefficient of Emission. Experiments on radiation such as the above are complicated by the fact that the greater portion of the loss of heat is due to convection currents in the air round the hot body and not to radiation. Dulong and Petit made observations on the rate of loss in a vacuum; in this case the heat lost is much less than when there is air or any other gas round the hot body. We may use the above experiment in the following way to calculate the loss of heat from the hot body. Let m be the mass of water in the vessel including the water equivalent of the vessel, let the temperature be $t_1^\circ \text{C.}$ and after 1 minute let it be $t_2^\circ \text{C.}$ Then the heat lost in 1 minute is $m(t_1 - t_2)$, and the heat lost per second or the rate of loss of heat is found by dividing this by 60. If we again divide by the area of the surface we can get what we may call the coefficient of emission of the surface at the given temperature, i.e. the amount of heat lost per second per square centimetre of surface under the given conditions.

Now it is found that this coefficient of emission depends only on the nature of the surface itself and on the temperatures of the surface and of the space into which it is radiating its heat.

We may fill the calorimeter with some other liquid, oil say; at a given temperature it radiates out heat at the same rate at which it radiated when filled with water. The fall in temperature is different in the two cases; the heat emitted, which will depend on the mass and specific heat of the liquid, is the same. Thus, suppose that the mass of the water is m grammes, that of the liquid M grammes, that it takes the water t seconds to cool through 5°C. from 70°C. to 65°C. and

that it takes the liquid T seconds to cool through the same range: let C be the specific heat of the liquid. The loss of heat from the water is $m \times 5/t$ units per second, that from the liquid when cooling through the same range is $M \times 5 \times C/T$ units per second. It is found that, omitting for the present the small correction due to the capacity for heat of the calorimeter, these two quantities of heat are equal, the heat emitted per second at a given temperature is the same in the two cases.

172. Specific Heat, Method of Cooling. The law just stated which can be verified by using a liquid of known specific heat may be made the basis of a method of determining the specific heat of a liquid. The method is known as the method of cooling.

EXPERIMENT (56). *To determine the specific heat of a liquid by the method of cooling.*

Take the calorimeter described in Section 170. Place in it a certain mass m grammes say of water, heated to about 70°C .— m may be about 100. Note the times taken by the water to cool through successive intervals of 5° —from 70° to 65° , 65° to 60° and so on down to say 30° . Empty out the water and place in the calorimeter a mass M of the liquid whose specific heat is required. The mass should be such as to fill the calorimeter to about the same extent as the water. Observe the times taken by the liquid to fall in temperature through the same intervals 70° to 65° etc. as the water. Let the time taken by the water to fall through some range, 65° to 60° say, be t seconds, that taken by the liquid over the same range T seconds. Then the amount of heat emitted per second by the water is $5m/t$, that emitted by the liquid is $5M \cdot C/T$, and these two quantities are equal.

$$\text{Thence} \quad \frac{5m}{t} = \frac{5MC}{T},$$

$$\text{or} \quad C = \frac{m}{M} \cdot \frac{T}{t}.$$

***173. Prevost's theory of exchanges.** We have seen that the amount of heat radiated from a body per

second diminishes as the temperature of the body approaches that of its enclosure. Two theories have been propounded to account for this. It may be that the presence of a second body at the same temperature as itself actually stops the radiation from the first body, while the presence of a cold body induces greater radiation; or it may be that the loss of heat is a differential effect, depending upon the difference between the amount of radiant energy emitted by the body and the amount it receives from the other body. This last is Prevost's theory of exchanges. According to it, the radiation emitted from a body depends upon its temperature and the nature of its surface, the radiation which falls on it depends on the temperature and surface of neighbouring bodies: if there is no change of temperature, it is not because the body has ceased to radiate but because the amount radiated is just equal to the amount absorbed from other bodies. Thus when as in Experiment 49 (c) an air thermometer and a hot ball are placed in the foci of two mirrors, the ball radiates to the air thermometer and the thermometer to the ball, the temperature of the thermometer rises because it receives from the ball more radiant energy than it emits; if a piece of ice be substituted for the hot ball, the temperature of the thermometer falls because it now emits more energy than it receives.

Or again, consider an enclosure E , fig. 82, containing a body A ; let the temperatures of the body and the enclosure be the same. Suppose also for simplicity that the bodies and the walls of the enclosure absorb all the incident radiation. The body is radiating out energy to the enclosure and receiving radiation from it; these two amounts are equal, hence the temperature is stationary. Now introduce a second body B at a different—say a lower—temperature. B rises in temperature, A and the walls of the enclosure fall. Part of the radiation from A which did fall on the enclosure and was balanced by radiation received from the enclosure now falls on B ; B being colder than A is radiating out less energy per

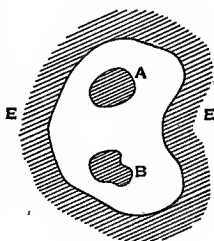


Fig. 82.

unit of area than A , it returns therefore to A less than it receives, it becomes warmer while A is cooled. Similar results follow for the radiation between B and the walls of the enclosure, and this continues until the temperature is equal throughout.

If the surfaces be not perfectly black, the argument will need a little modification to allow for the reflections which take place, but the principles are the same.

***174. Prevost's theory and the relations between the absorbing and emissive powers of a surface.** We may shew from Prevost's theory that the absorbing and emissive powers of a body must be equal in the following way.

Consider a black body, such as the blackened bulb of a thermometer, fig. 83, surrounded by a surface which reflects some of the incident radiation and absorbs the rest. Suppose the temperature to remain the same throughout, and let the outer surface reflect, say 30 per cent., of the incident radiation from the black body and absorb the remaining 70 per cent. Then out of every hundred units of energy which leave the black body, 30 are reflected to it from the enclosure; and these 30 are again absorbed by it. Thus out of the original hundred, 70 units leave the black body and enter the enclosure; if this were all the temperature of the enclosure would rise, that of the central body would fall. But the temperatures remain constant; thus the enclosure must in the same time lose 70 units which must enter the black body. These 70 units are radiated by the enclosure to the black body. Thus a surface which absorbs 70 per cent. of the energy absorbed by a black surface at the same temperature, emits 70 per cent. of the energy emitted by that black surface. The emissive and absorbing powers are equal, and the sum of either, together with the reflecting power, is equal to the emissive power of the black surface.

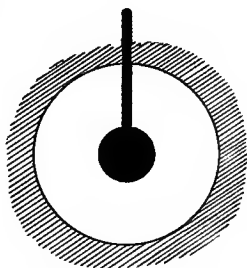


Fig. 83.

EXAMPLES.**RADIATION.**

1. How would you shew that radiant heat is refracted similarly to light? Explain the action of a glass fire-screen.

2. A red-hot ball of iron looks equally bright in all its parts and at whatever distance from the eye it be placed. Explain these facts.

3. A vessel containing hot water is placed on a table: explain by what means the vessel and water fall in temperature.

4. Shew that it follows from Prevost's theory of exchanges that the absorbing powers of two surfaces are proportional to their radiating powers; how may this be also verified by experiment?

5. State the difference of behaviour of plates of rock-salt, alum and glass, with respect to the transmission of radiant energy. Describe how the energy radiated from a black body varies as its temperature is continuously raised.

6. What experiments would you undertake to determine (a) the radiating, (b) the absorbing power of a surface? How are they connected?

7. Describe the thermopile. How would you prove experimentally the law connecting the intensity of the radiation at a point with its distance from the source of heat?

8. State Prevost's theory of exchanges and explain the arguments upon which it rests.

A calorimeter is hung up within an outer vessel and it is desired to keep it as far as possible from losing or gaining heat. Explain the advantages, if any, of silver-plating (a) the outside of the calorimeter, (b) the inside of the outer vessel.

9. The bulbs of two identical thermometers are coated, the one with lampblack, the other with silver; compare their readings (1) when in a water bath in a dark room, (2) when in the sun, (3) when exposed on a clear night; explaining why they do not agree on all these occasions.

CHAPTER XIII.

THE MECHANICAL EQUIVALENT OF HEAT.

***175. Joule's experiments on the equivalence of heat and energy.** These experiments have already been referred to. The arrangement of the apparatus in his last experiments is shewn in figs. 84 and 84*a* which are taken from Joule's paper. Fig. 84*a* gives the calorimeter and the paddles. The calorimeter is shewn again in fig. 84.

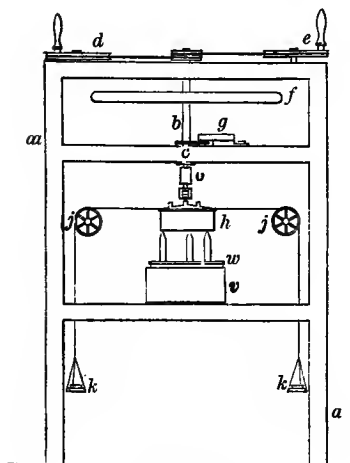


Fig. 84.

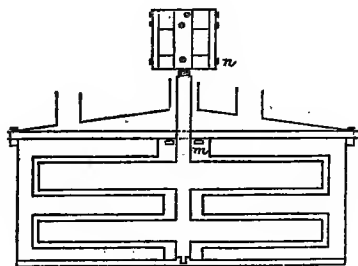


Fig. 84*a*.

The general plan of the experiment is to heat the water by churning it up with the paddles, to measure the work done in the heating and the heat produced in the water and calorimeter, and to find the ratio of the two. In order to calculate the work done, the following arrangement due to Hirn was employed. The calorimeter is mounted so as to be free to turn round a vertical axis. The paddles are carried on a vertical axis coinciding with that about which the calorimeter can turn. On this axis, bc , are two pieces of box-wood o and n which help to prevent the conduction of heat from the bearing at c down to the water. In the latter experiments the calorimeter was supported on a hollow cylindrical vessel w which floated in water in the vessel v , and thus took the pressure off the bearings. The axle bc carrying the paddles and the horizontal flywheel f can be made to rotate by turning the wheels d and e . At g there is a counter which indicates the number of rotations of the axle. If everything be free the friction between the water and the calorimeter will carry the calorimeter round with the paddles, the water will not be churned and therefore it will not be heated.

By applying force to the calorimeter it can be prevented from rotating; the water will be made to move round inside relatively to the calorimeter and be heated in consequence of the work that is done by the forces between the water and the calorimeter. In Joule's experiments the forces restraining the motion of the calorimeter were the tensions of two thin strings which rested in a horizontal groove round the calorimeter and passing over two light pulleys j, j carried weights k, k . These weights could move up and down near two scales by which their positions could be fixed, and were adjusted until they remained stationary, while the shaft and paddles revolved at some suitable uniform speed. If the speed quickened a little the weights rose, if it slowed they sank, and by watching the weights a uniform rate of working could be insured. Thus the tension in the string required to keep the calorimeter at rest is measured. Let us call the tension of each string W , it will be equal to the weight in either scalepan together with the weight of the pan.

Now in the experiment the water is made to rotate by holding the calorimeter fixed and spinning the axle and paddles. We could get the same effect by holding the paddles fixed and allowing the same force W to act on either string and cause the calorimeter to turn, the number of rotations per second and the work done would be the same as in the actual experiment, the weights k, k would move down uniformly and the string unwind off the groove round the calorimeter. Now in this case, if a be the radius of the calorimeter, in a single turn each weight would descend through a distance equal to the circumference or $2\pi a$, and in n turns it would go through n times this distance. The work done therefore by the two weights would be

$$2W \times n \times 2\pi a \text{ or } 4n\pi a W,$$

and the quantities involved in this can be accurately measured. It remains now to determine the heat generated by this work. For this purpose it is necessary to know the mass M of the water in the calorimeter, the water equivalent m of the calorimeter and paddles, and the rise in temperature t of the water. The heat generated then is $(M + m)t$. Thus the ratio of the work done to the heat generated is

$$4n\pi a W / (M + m)t.$$

Corrections will be required for the loss of heat by radiation, the friction at the pullies, the work done in raising the velocity to its steady value and various other points. It follows from Joule's experiments that this ratio which is the amount of work required to produce 1 unit of heat is constant. It is called "Joule's equivalent" and is denoted by J .

In one series of Joule's experiments the average values were

$$M + m = 84280 \text{ grains}$$

$$2W = 18229 \text{ grains}$$

$$2\pi a = 2.774 \text{ feet}$$

$$n = 4870$$

$$t = 3^{\circ}.768 \text{ Fah.}$$

On substituting these numbers in the formula we find for the ratio the value 775. (The actual result given in Joule's

Tables for this series of experiments is 774.57.) This result therefore is the number of foot-grains of work required to raise the temperature of 1 grain of water 1° Fah., or the number of foot-lbs. required to raise the temperature of 1 lb. of water 1° Fah. at $54^{\circ}.7$ Fah. which was the average temperature of this series of experiments. The result given by Joule himself at the conclusion of his last paper as the value for the mechanical equivalent is 772.55 ft. lbs. at 60° Fah. in the latitude of Greenwich.

Now in this result 1 degree of temperature is a degree as reckoned on his mercury-thermometer. This will differ from a degree on the air thermometer by a small amount. Prof. Rowland introduced the correction for this into Joule's work.

The result of introducing these corrections is to make Joule's value come to 776.75 foot-pounds at 15° C., or if we take all Joule's published values instead of the 772.55 of this last series we find as Joule's mean result 779.17.

Expressed in the metric system in terms of the work done in raising 1 gramme through 1 centimetre and the heat necessary to raise 1 gramme 1° C. this number reduces to 427.50.

Prof. Rowland adopts 776.75 as the result of Joule's work, and this in metric units comes to 426.75. Joule's experiments were repeated in 1878 by Rowland who gives as his result 427.52, while Mr Griffiths using a method depending on the heat produced in a wire by an electric current finds 428.4.

Thus we may take the value 778 or 779 foot-pounds of work as representing very approximately the amount of work necessary to raise the temperature of 1 lb. of water 1° Fah. Since a degree centigrade is $\frac{9}{5}$ of a degree Fahrenheit the value of J , employing centigrade degrees and pounds, is $779 \times \frac{9}{5}$ or 1402 foot-pounds. Expressed in ergs the value is 4.194×10^7 .

***176. Determination of J by friction of metal on metal.** In some of Joule's experiments the value of J was found by the friction of metal on metal. One experiment by this method can be carried out by means of the apparatus

shewn in fig. 85 (for practical details see Glazebrook and Shaw, *Practical Physics*, p. 290).

A cast-iron cup *C* is secured to a vertical axle working in bearings and driven by a band over the horizontal pulley *A*.

Inside this cup there is a conical brass cup, shewn in section at the side, separated from the iron cup by a layer of cork to which it is firmly secured; a second brass cup fits in this and is carried round by the friction between the two when the axle is rotated. A large wooden pulley *D* is attached to the upper cup and a string passes round this and over a fixed pulley and carries a weight *P*. The outer cup is then made to rotate until the friction between the two is just sufficient to begin to

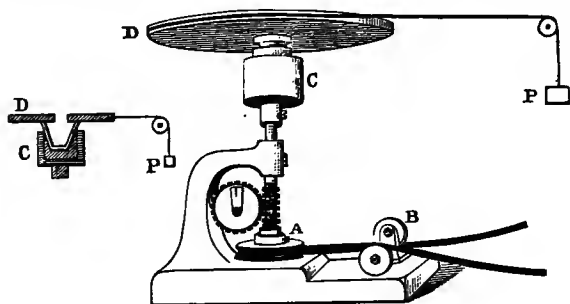


Fig. 85.

cause the inner cup to turn and to raise the weight *P*; the work spent in overcoming this friction is measured as in the previous experiment. The inner cup contains a known mass of water and the friction heats this water. The heat produced can be calculated in terms of the mass of water, the water equivalent of the calorimeter and the rise in temperature. Knowing the quantity of heat produced and the work expended in its production we can find a value for *J*.

***177. Work done by the expansion of a gas.** Suppose that a gas is contained in a cylinder with a piston under pressure. Reduce the pressure in some way as by suddenly lifting a weight off the piston. The gas expands and

in expanding does work in overcoming the pressure which remains on the piston. To do this work energy is required, and in consequence the gas is cooled down by the expansion. This may be easily shewn by compressing air into a vessel, leaving it to acquire the temperature of the air, then opening a tap and allowing the air to escape against the face of a thermopile or a delicate thermometer. But now when the expansion has ceased the air occupies a greater volume than before, the average distance between its particles will be greater than previously, and if there are any attractive forces between the air particles work will have been done in overcoming this attraction: part of the cooling effect will be due to this, part only to the external work done in raising the piston. If, on the other hand, there were repulsive forces between the particles, these forces would help to overcome the external force and therefore the cooling would be less than we should calculate from the work done in overcoming the pressure.

Joule shewed that neither of these effects took place in air, or the other permanent gases, to any appreciable extent; there are therefore neither attractive nor repulsive forces acting continuously on the air molecules; when the air is allowed to expand, the cooling produced is wholly accounted for by the external work done.

Joule's apparatus consisted of two copper receivers *R* and *E*, fig. 86. These were connected by a tube *D* with a stopcock, and placed inside a tin vessel which was filled with water; this vessel contained about $16\frac{1}{2}$ lbs. of water and each of the copper receivers would hold about 134 cubic inches. One vessel *E* was exhausted, the other filled with air at a pressure of about 22 atmospheres, and the temperature of the surrounding water was carefully noted.

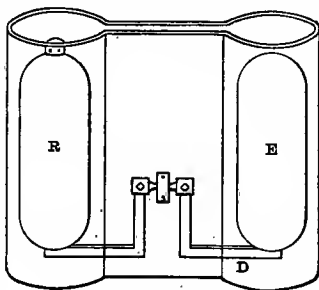


Fig. 86.

The stopcock was then opened and the air allowed to pass into the empty receiver. The water was then stirred and the

change in temperature was found to be unappreciable. The vessel from which the gas escapes loses heat, the other vessel gains it, but the loss and the gain are exactly equal, and so, on the whole, after stirring there was no change of temperature. In another experiment the two receivers were put into separate vessels and it was verified directly that the loss of heat in the one was equal to the gain in the other. According to these results then, if a gas expands, all the energy derived from its fall of temperature is used in overcoming the external pressure, none is needed to do internal work¹.

Suppose now that a gas contained in a cylinder with a piston is allowed to expand from a volume v to a volume v' , let A be the area of the piston, x the distance it moves and p the pressure, then as the piston moves, the volume increases by Ax , thus $v' - v = Ax$.

The force on the piston is Ap and the piston on which this force acts moves a distance x , hence the work done is

$$Ap x \text{ or } p \times Ax.$$

Now
$$p \times Ax = p \times (v' - v).$$

Thus in this case when a gas increases from volume v to volume v' , at a constant pressure p , the work it does is

$$p(v' - v).$$

It can be shewn that this expression holds generally, and not merely in the case in which the gas is in a cylinder.

***178. Mayer's method of determining J.** It has already been pointed out that the specific heat of a gas at constant pressure differs from that at constant volume, and the reason is now clear, if in the first case heat is needed to supply energy sufficient to overcome the external pressure, in addition to raising the temperature, in the second it is only required to raise the temperature; and since by the last experiment the whole difference between these two amounts of heat is used in doing external work, we can get a relation between that heat and the work done. This was Mayer's method of finding J ,

¹ In more elaborate experiments Joule and Thomson have shewn that this is not absolutely true, but it is a very close approximation.

but he did not shew that his argument was legitimate, because he omitted to prove that none of the energy was used in doing internal work against internal forces. Joule supplied this proof and the argument is now legitimate.

Let v be the volume of a unit of mass of gas, and T the absolute temperature of the same, p its pressure, then in raising its temperature 1° at constant pressure v expands by the amount v/T and the work done is pv/T . Let c_p and c_v be specific heats at constant pressure and constant volume. Since the mass of gas is unity, c_p is the heat which must be applied to raise its temperature 1°C. at constant pressure, c_v at constant volume. Thus the amount used in doing external work is the difference between these or $c_p - c_v$.

The mechanical equivalent of this is found by multiplying by J , it is therefore

$$J(c_p - c_v),$$

and this is equal to the work done; thus

$$J(c_p - c_v) = \frac{pv}{T},$$

hence

$$J = \frac{pv}{T(c_p - c_v)}.$$

Now v is the volume of a unit of mass, it is therefore equal to $1/\rho$ where ρ is the density. Writing this we have

$$J = \frac{p}{\rho T(c_p - c_v)}.$$

The value of c_v is not known with sufficient accuracy to make this a very good method of finding J , we may however if we take air as the gas at the standard pressure one atmosphere and temperature zero, using the ordinary values, put

$$p = 1.013,000 \text{ dynes per sq. cm.}$$

$$\rho = .001276 \text{ grammes per c.cm.}$$

$$T = 273$$

$$c_p = .2375$$

$$c_v = .1684.$$

And from these numbers we find

$$J = 4.21 \times 10^7 \text{ ergs per gramme degree centigrade.}$$

***179. Graphical representation of work done by a gas in expanding.** We have just seen that if a gas changes its volume at constant pressure the work done in expansion is measured by the pressure multiplied by the change of volume.

Now let us represent the volume as before in Section 118 by horizontal lines parallel to OA , the pressure by vertical lines parallel to OB . Then, if the pressure remains constant, the curve on the diagram representing the relation between it and the volume is a horizontal straight line such as P_1P_2 fig. 87, in which ON_1 , ON_2 represent two volumes v_1 , v_2 and N_1P_1 , N_2P_2 the corresponding pressures. Then N_1N_2 is the change in volume and the parallelogram $P_1N_1N_2P_2$ measures the work done. A similar construction gives the work done in the more general case when the pressure is not constant, for let PQ , fig. 88, represent the relation between the pressure and the volume, and let the volume change from ON_1 to ON_2 by a very small amount N_1N_2 . The change in pressure will be very small and the pressure may be represented as either P_1N_1 or P_2N_2 , being something between these values. The work done will lie between the rectangle P_1N_2 and the rectangle P_2N_1 ; if N_1 and N_2 are very close, it may be represented by the curvilinear area $P_1N_1N_2P_2$. Proceeding thus we see that the work done in any finite change of volume such as LM , is given by the area $PLMQ$, where LP and MQ represent the pressures corresponding to volumes OL and OM .

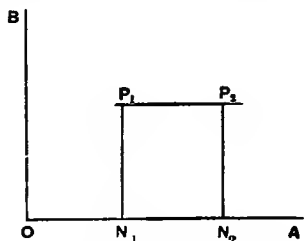


Fig. 87.

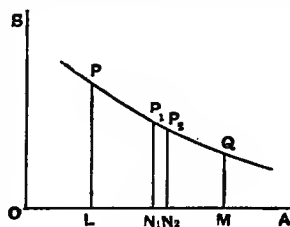


Fig. 88.

This method of measuring work is made use of in the indicator diagrams of steam and other engines

EXAMPLES.**HEAT AND ENERGY.**

1. What do you understand by the statement that Heat is a form of Energy and what are the grounds for this statement?

2. What do you understand by a diagram of energy and by an isothermal line on such a diagram? Shew how to calculate the work done during a series of changes represented on such a diagram.

3. What is meant by the critical temperature, pressure, and volume of a gas? How can these be determined experimentally?

Draw the isothermals of a substance at temperatures above and below the critical temperature.

4. What is meant by the mechanical equivalent of heat, and how has it been determined?

If the work a man does in running upstairs is half spent in producing heat and half in doing mechanical work, by how much will the temperature of a man be raised who runs up a flight of stairs 20 feet high, assuming the specific heat of the man to be the same as that of water, and that no heat is lost by radiation or other causes?

5. The combustion of a gramme of coal produces 8000 units of heat. If an engine employs in pumping water one-tenth of the energy supplied to its boiler by the combustion of coal, find how much coal must be burnt in order to enable it to raise 5000 litres of water to a height of 10 metres.

6. Explain how the mechanical equivalent of heat may be calculated from a knowledge of the specific heats of air at constant pressure and constant volume.

Note any assumptions made in the calculation and state how they may be justified.

7. If the mechanical equivalent of heat be 779 foot-lbs. Fahrenheit, from how high must 10 lbs. of water fall to raise its temperature 1° Centigrade?

8. The Falls of Niagara are 165 feet high. Find by how much the temperature of the water will be increased by the fall, supposing that the whole kinetic energy acquired by the water in its fall is converted into heat.

9. A meteorite weighing 2000 kilogrammes falls into the sun with velocity of 1000 kilometers per second. How many calories will be produced by the collision?

(Mechanical equivalent of 1 calorie = 4.19×10^7 ergs.)

10. The mechanical equivalent of heat is 1390 foot-pounds, and it is found that a pound of coal produces by its consumption 8000 units of heat. An engine employs 10 per cent. of the heat supplied to its boiler in drawing a weight up a slope of 30° . Find the coal that must be burnt to draw a weight of 100 lbs. for 1 mile along the slope.

11. A quantity of mercury is allowed to fall a distance of 7.79 feet. If all the mechanical energy lost by the fall is converted into heat in the mercury, calculate how much hotter the mercury will be after the fall, taking the specific heat of mercury as $\frac{1}{3}$.

12. In certain experiments on boring cannon it was ascertained that one horse working for 2 hours 30 minutes raised by 180° F. the temperature of a mass equivalent in capacity for heat to 26.58 lbs. of water. Assuming that a horse does 30,000 foot-pounds of work per minute, deduce the value of the mechanical equivalent of heat.

13. Assuming the combustion of a lb. of coal produces 8000 units of heat, how many lbs. of coal must be consumed to produce the equivalent of the work done in raising a weight of 12 stone to a height of 15,000 feet?

14. The combustion of 1 lb. of coal raises the temperature of 100 gallons of water through 4.4 degrees; find the mechanical equivalent of this quantity of heat.

EXAMINATION QUESTIONS.

I.

1. Give an outline of the experiments which led Rumford to believe that Heat was not a material substance.
2. Distinguish between heat and temperature. Define temperature and explain the analogy between it and hydrostatic pressure.
3. Describe the construction and graduation of a mercurial thermometer. What are the fixed points of the thermometer scale?
4. Shew how to reduce the readings of a Fahrenheit thermometer to the Centigrade and Réaumur scales.
5. Describe some forms of maximum and of minimum thermometers.

II.

1. Define the terms unit quantity of heat, capacity for heat, specific heat; and shew from your definition that the quantity of heat required to raise a mass m of specific heat c from t° to T° is $mc(T - t)$.
2. Explain how to determine specific heat by the method of mixture. How would you allow for the heat absorbed by the Calorimeter?
3. Define Latent heat, and explain some method of finding the latent heat of ice.
4. Describe some form of ice Calorimeter.
5. Distinguish between evaporation and boiling, and shew that the boiling point of water depends on the pressure.

III.

1. Define the latent heat of steam.

On passing 10 grammes of steam into 100 grammes of water at 15° the temperature rises to 71° ; find the latent heat of steam.

2. Define the coefficients of linear and of cubical expansion of a substance and shew that the latter is three times the former.

Explain some method of measuring the coefficient of linear expansion of a metal.

3. Describe a method of finding the coefficient of expansion of a liquid.

4. Distinguish between the apparent and the real expansion of a fluid and shew that, if α be the real coefficient, and α' the apparent, then $\alpha - \alpha'$ is the coefficient of expansion of the containing vessel.

5. Describe some method of finding the absolute coefficient of expansion of mercury.

In correcting a barometer reading would you use the absolute or the relative coefficient of expansion?

6. State the laws connecting the pressure, volume and temperature of a gas, and describe experiments to prove them.

IV.

1. Explain what is meant by the absolute zero of the air thermometer, and by absolute temperature.

2. Describe an apparatus to prove that air at constant pressure expands in volume for each degree centigrade by $1/273$ of its volume at 0°C .

3. How would you prove experimentally that the pressure of a gas at constant volume is proportional to the absolute temperature?

4. Assuming Boyle's Law and Charles' Law prove that the pressure of a gas at constant volume increases by a given fraction of the pressure at 0° for each rise of temperature of 1° .

5. Describe a form of air thermometer suitable for the measurement of a high temperature such as the boiling point of sulphur.

6. Distinguish between a gas and a vapour and state and explain Dalton's Law as to the pressure of mixed vapours.

V.

1. Shew that if a gas expand at constant pressure p from volume v to volume v' then the work done is $p(v' - v)$. Explain why the specific heat of a gas at constant pressure is greater than the specific heat at constant volume.

2. Define the dew-point, and explain some methods of determining it. Shew how to find the pressure of the aqueous vapour present in the air from a knowledge of the dew-point.

3. A quantity of dry air measures 1000 cubic centimetres at 10°C . and 760 mm. pressure. If the same air is heated to 30° and saturated with moisture at that temperature, what must be the pressure in order that the volume may remain unchanged? The saturation pressure of aqueous vapour at 30° is 31.5 mm.

4. Explain what is meant by the critical temperature of a gas. What processes are necessary in order to liquefy oxygen? Distinguish between gases and vapours.

5. Describe the thermopile. How would you prove that if an electric current be produced by applying heat to a junction of two metals, then that junction is cooled if an electric current be made to traverse it?

6. Distinguish between radiation, convection, and conduction of heat; and describe an experiment to compare the conductivities of two bars of metal, explaining carefully why it is necessary to wait till a steady condition is reached before making the measurements.

ANSWERS TO EXAMPLES IN HEAT.

CHAPTER III. (Page 27.)

- | | | |
|--|---|----------|
| 4. $44^{\circ}\cdot 4\text{ C.}, 200^{\circ}\text{ C.}$ | 6. $37^{\circ}\cdot 7\text{ C.}, 4^{\circ}\cdot 4\text{ C.}, -17^{\circ}\cdot 7\text{ C.}, 36^{\circ}\cdot 6\text{ C.}$ | |
| 9. $30^{\circ}\text{ C.}, -17^{\circ}\cdot 7\text{ C.} - 30^{\circ}\text{ C.}, 212^{\circ}\text{ F.}, 14^{\circ}\text{ F.}, -22^{\circ}\text{ F.}$ | | 11. 144. |

CHAPTER IV. (Page 49.)

- | | | |
|--|---|-----------------------------------|
| 1. $25^{\circ}\cdot 65, \cdot 064, 449^{\circ}\cdot 6\text{ Fah.}$ | 2. $116^{\circ}\cdot 37.$ | 3. $99^{\circ}\cdot 43\text{ C.}$ |
| 4. $95, 118^{\circ}\cdot 7\text{ grms.}$ | 6. 20° C. | 7. $6^{\circ}\cdot 8\text{ C.}$ |
| 8. $\cdot 638.$ | 9. $48^{\circ}\cdot 9\text{ C.}$ | 10. 80. |
| 11. $71^{\circ}\cdot 5\text{ C.}$ | 12. $\cdot 0869.$ | 14. $\cdot 446\text{ lbs.}$ |
| 15. $\cdot 0996.$ | 16. $83^{\circ}\cdot 3.$ | 17. $52^{\circ}.$ No. |
| 21. (a) $\frac{59}{9}\text{ lbs.}$ are melted, | (b) ice is melted and raised to $16^{\circ}\cdot 2\text{ C.}$ | |
| 22. $938^{\circ}\cdot 8\text{ C.}$ | 23. 530. | 24. $80^{\circ}\cdot 9.$ |
| | | 25. $12^{\circ}\cdot 32.$ |

CHAPTER V. (Page 72.)

- | | | |
|--|-----------------------------------|----------------------------------|
| 1. $\cdot 9995\text{ c. ft.}, 1\cdot 0045\text{ c. ft.}$ | 2. $2\cdot 000534\text{ litres.}$ | 3. $7^{\circ}\cdot 733.$ |
| 4. $\cdot 0000167.$ | 5. $\cdot 198\text{ ft.}$ | 6. 80° C. |
| 7. $\cdot 131\text{ cm.}$ | 8. $\cdot 928\text{ metre.}$ | 9. $29^{\circ}\cdot 7\text{ C.}$ |
| 10. $\cdot 04212\text{ inch.}$ | 11. $50\cdot 0058\text{ sq. cm.}$ | 12. $\cdot 0306\text{ yd.}$ |
| 13. $\cdot 991\text{ grm.}$ | 15. $\cdot 184\text{ inch.}$ | 16. $\cdot 0000185.$ |

CHAPTER VI. (Page 94.)

- | | | |
|-----------------------------|---------------------------------|----------------------------------|
| 1. $\cdot 000192.$ | 2. $5\cdot 35\text{ grms.}$ | 5. $\cdot 000152, \cdot 000013.$ |
| 6. $752\cdot 65\text{ mm.}$ | 7. $753\cdot 25\text{ mm.}$ | 8. $\cdot 0000076.$ |
| 9. $\cdot 0000095.$ | 10. $29\cdot 42\text{ inches.}$ | |

CHAPTER VII. (Page 114.)

- | | | |
|---------------|-------------------------------|-----------------|
| 2. 1330 mm. | 3. 345·2 c. in., 577·5 c. in. | 5. 19·43 c. ft. |
| 6. 10 c. ft. | 9. 41·25 c.cm. | 10. 4 c.cm. |
| 11. 131°·7 C. | 12. ·877 grm. per litre. | 13. 32·74 c.cm. |
| 14. ·00368. | 15. 50° C. | |

CHAPTER VIII. (Page 121.)

- | | |
|--------------------|-------------------|
| 8. 200160 ft. lbs. | 9. 20 hrs. 5 min. |
|--------------------|-------------------|

CHAPTER IX. (Page 157.)

- | | | |
|---------------------------|---|----------------|
| 1. 90·7 c.cm. | 9. ·0958 grm. | 10. ·0492 grm. |
| 12. ·634 atmo.
to 288. | 14. (1) not at all, (2) changed in ratio of 283 | |
| | 15. 12·7 mm. | 16. ·101 grm. |
| 17. 1318 grms. | 18. 99°·43 C. | |

CHAPTER XI. (Page 176.)

- | | | |
|-----------------|-------------|---------------|
| 2. 537000 grms. | 5. 2492000. | 6. 105°·39 C. |
| 8. 1080000. | 11. 15. | |

CHAPTER XII. (Page 220.)

- | | | |
|---------------------------|-------------------------|---------------|
| 4. $\frac{1}{4}$ of 1° C. | 5. 1460 grms. | 7. 1402 ft. |
| 8. 0°·1177 C. | 9. 2387×10^7 . | 10. 0·237 lb. |
| 11. 0°·033 Fah. | 12. 985 ft. lbs. | 13. 0·225 lb. |
| 14. 3·138 ft. lbs. | | |

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